O Método de Newton para sistemas não-lineares e otimização, e comentários sobre a convergência

Seminários Poincarè

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Universidade Federal do Paraná

20 de Novembro de 2015

Zeros de uma função real

$$f(x) = 0, \qquad f : \mathbb{R} \to \mathbb{R}$$

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$$f(x) = (x^2 - 1)(x^2 - 4) + \sin(3x - 3)$$

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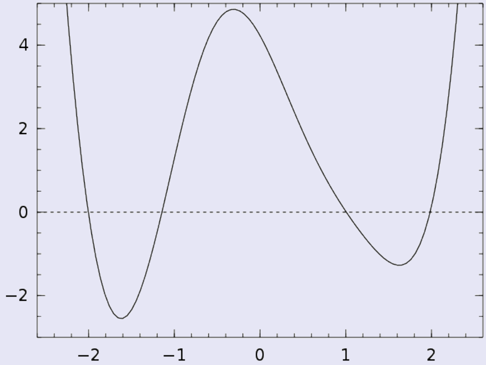

Aproximação linear

$$f(x) = f(a) + f'(a)(x - a) + E(x)$$

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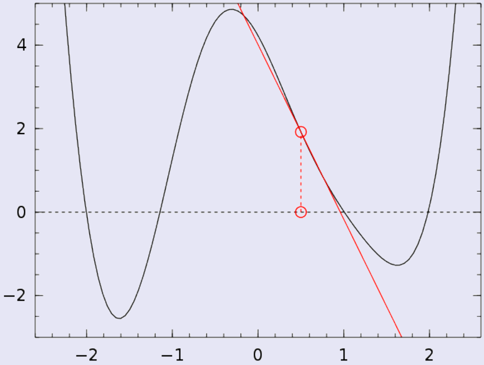

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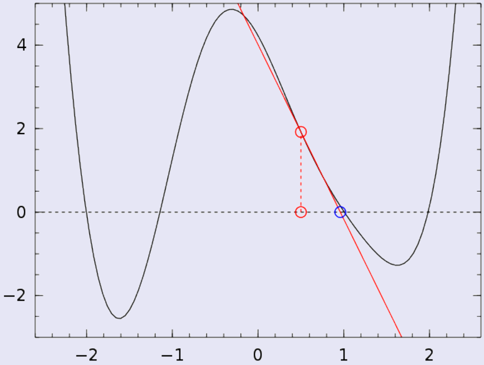

O zero da função afim

$$L(x) = f(a) + f'(a)(x - a) = 0$$

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$$x = a - \frac{f(a)}{f'(a)}, \qquad f'(a) \neq 0$$

Método de Newton

Dado x^0, defina a sequência

$$x^{k+1} = x^k - \frac{f(x^k)}{f'(x^k)}$$

se $f'(x^k) \neq 0$.

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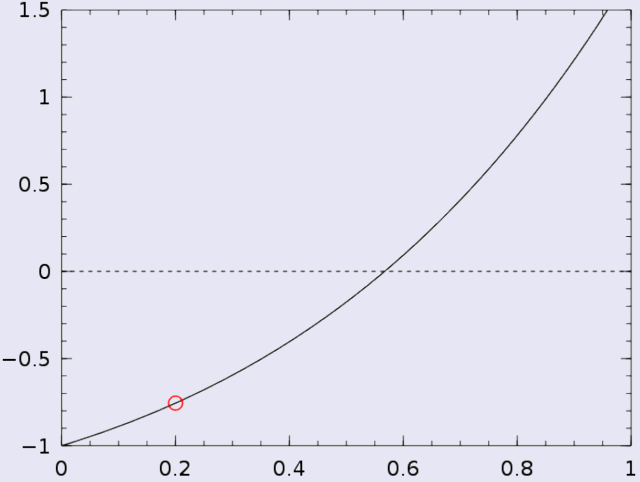
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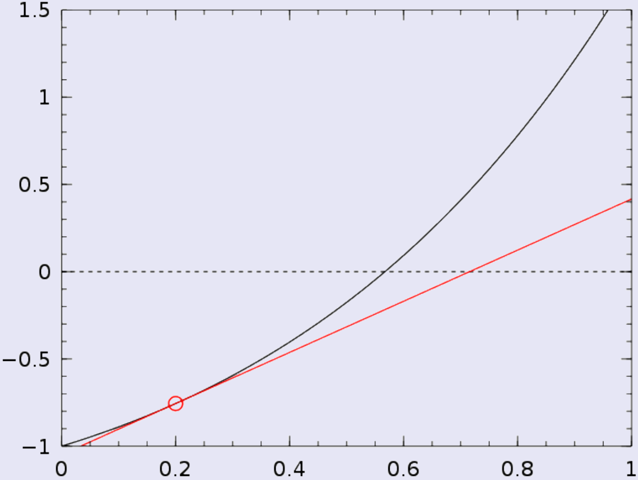
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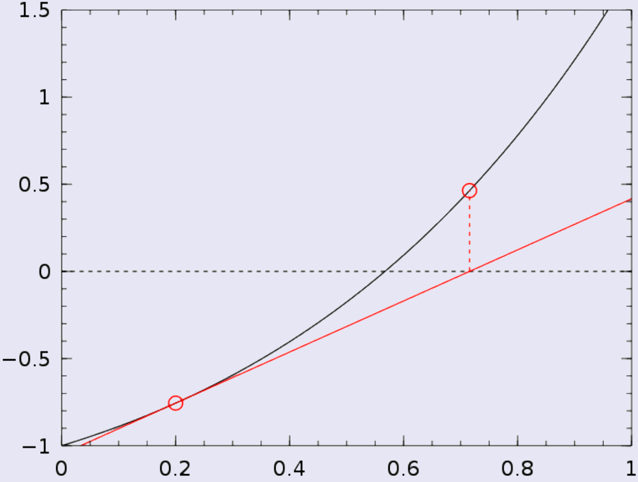
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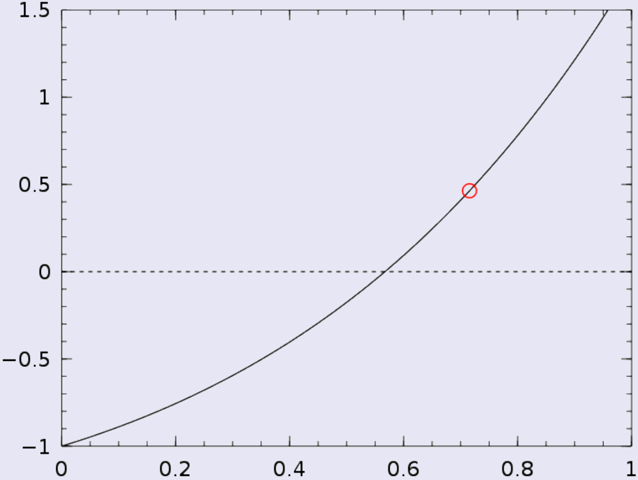
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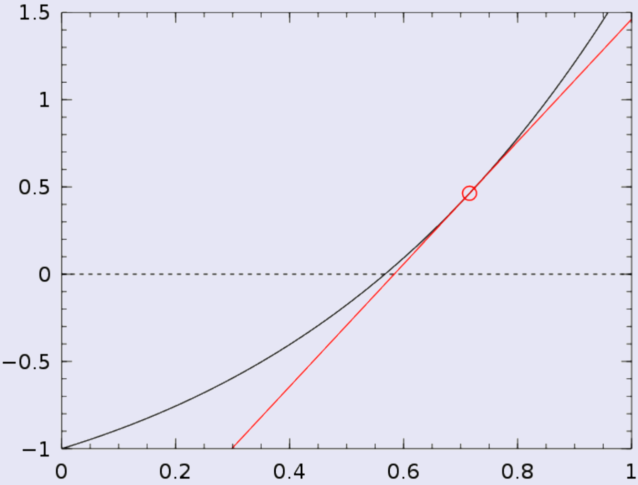
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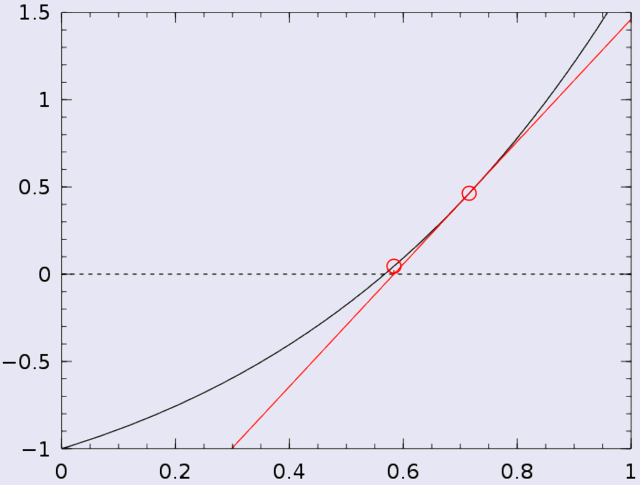
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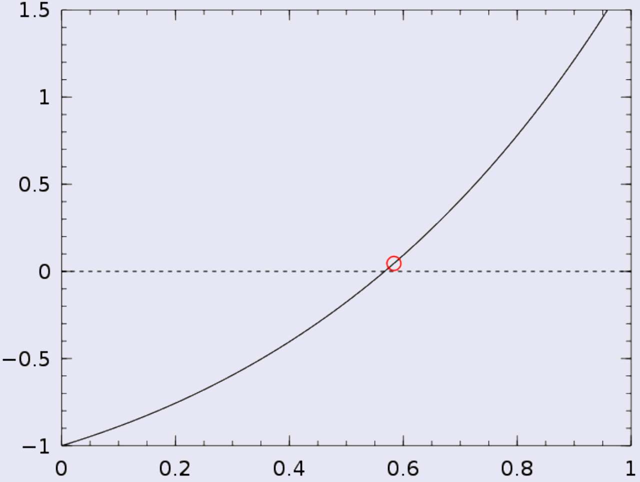
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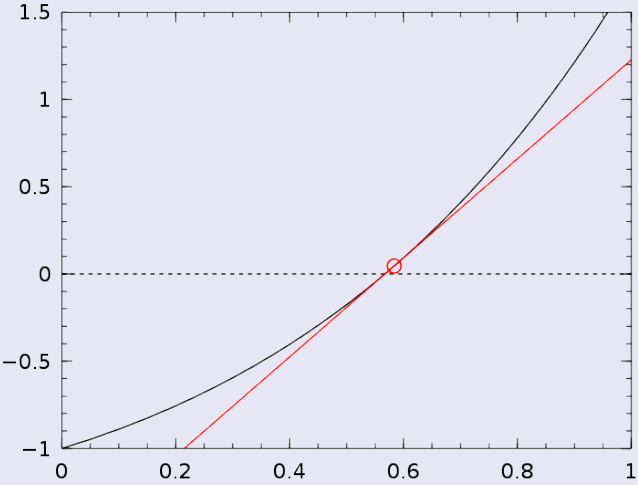
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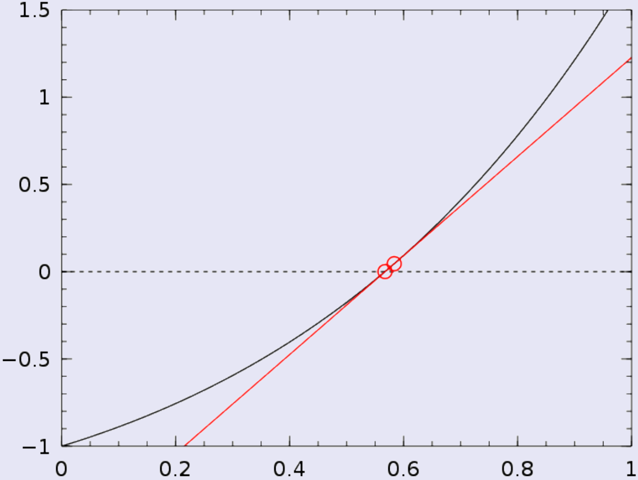
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Equações não-lineares

$$\begin{cases} x_1^2 + x_2^2 & = & 4 \\ x_1 x_2 & = & 1 \end{cases}$$

$$\begin{cases} x_1^2 + x_2^2 - e^{x_1 + x_2} & = & 1 \\ \sqrt{1 + x_1^2} + \dfrac{1}{x_2^2 + 1} & = & 1 \end{cases}$$

$$\begin{cases} x_1 x_2 x_3 & = & 1 \\ x_1 x_2 + x_1 x_3 + x_2 x_3 & = & 3 \\ x_1 + x_2 + x_3 & = & 3 \end{cases}$$

Equações não-lineares

$$F(x) = 0, \qquad F : \mathbb{R}^n \to \mathbb{R}^n.$$

$$\begin{cases} x_1^2 + x_2^2 & = & 4 \\ x_1 x_2 & = & 1 \end{cases}$$

$$F(x) = \begin{bmatrix} x_1^2 + x_2^2 - 4 \\ x_1 x_2 - 1 \end{bmatrix}.$$

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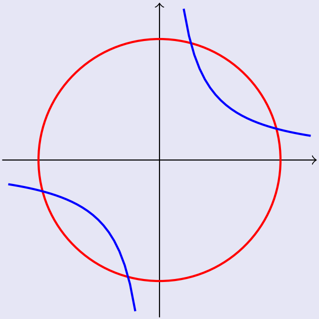

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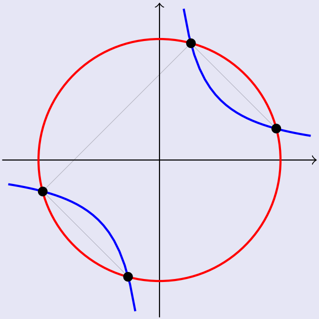

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$$F(x) = \begin{bmatrix} x_1^2 + x_2^2 - 4 \\ x_1 x_2 - 1 \end{bmatrix} \quad F'(x) = \begin{bmatrix} 2x_1 & 2x_2 \\ x_2 & x_1 \end{bmatrix} \quad a = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

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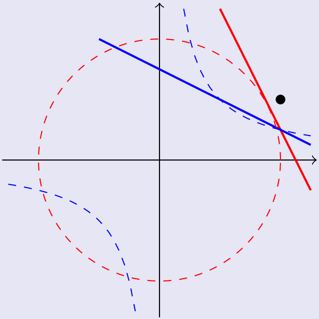
$$F(2,1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad F'(2,1) = \begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix}$$

$$L(x) = \begin{bmatrix} 4x_1 + 2x_2 - 9 \\ x_1 + 2x_2 - 3 \end{bmatrix}.$$

Equações não-lineares

$$\begin{cases} 4x_1 + 2x_2 = 9 \\ x_1 + 2x_2 = 3 \end{cases}$$



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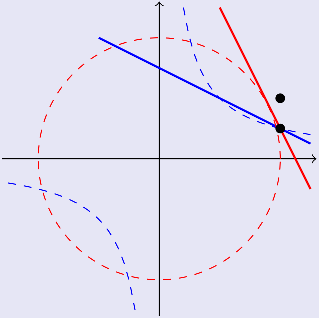

A solução do sistema linear

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$$x = a - [F'(a)]^{-1} F(a),$$

$F'(a)$ não singular.

O Método de Newton

Dado x^0, defina as sequências $\{d^k\}$ e $\{x^k\}$ por

$$F'(x^k)d^k = -F(x^k),$$

$$x^{k+1} = x^k + d^k,$$

supondo-as bem definidas.

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Sob **hipóteses** $F(x^k) \to 0$.

O Método de Newton

$$x^0 = (2.00000, 1.00000), \qquad F(x^0) = (1.00000, 1.00000)$$
$$x^1 = (2.00000, 0.50000), \qquad F(x^1) = (0.25000, 0.00000)$$
$$x^2 = (1.93333, 0.51667), \qquad F(x^2) = (0.00472, -0.00111)$$
$$x^3 = (1.93185, 0.51764), \qquad F(x^3) = (0.00000, -0.00000)$$

* Com 5 casas decimais

Otimização

$$\min f(x), \qquad f : \mathbb{R}^n \to \mathbb{R}, f \in C^2$$

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CN2 Se x^* é um minimizador local de f, então $\nabla f(x^*) = 0$ e $\nabla^2 f(x^*)$ é semidefinida positiva.

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CS2 Se $\nabla f(x^*) = 0$ e $\nabla^2 f(x^*)$ é definida positiva, então x^* é um minimizador local de f.

O Método de Newton para otimização

$$F(x) = \nabla f(x)$$

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O Método de Newton para otimização

$$\nabla^2 f(x^k)d + \nabla f(x^k) = 0.$$

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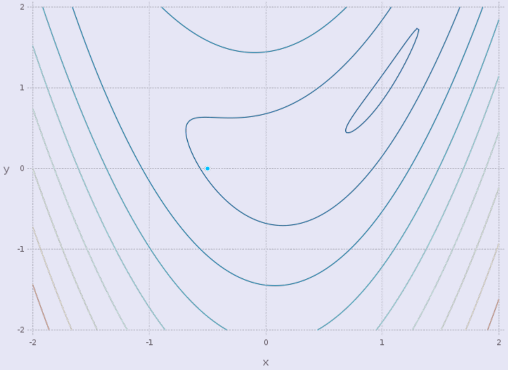
$$m(d) = \frac{1}{2}d^T\nabla^2 f(x^k)d + \nabla f(x^k)^T d + f(x^k).$$

$$d^k = \arg\min_d m(d).$$

$$x^{k+1} = \arg\min_x m(x - x^k).$$

O Método de Newton para otimização

$$f(x) = (1 - x_1)^2 + 4(x_2 - x_1^2)^2, \quad x^0 = (-0.500, 0.000)$$



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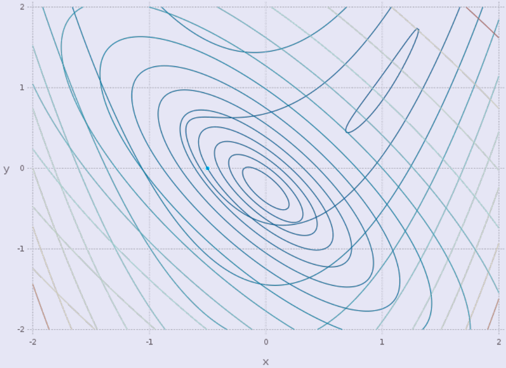

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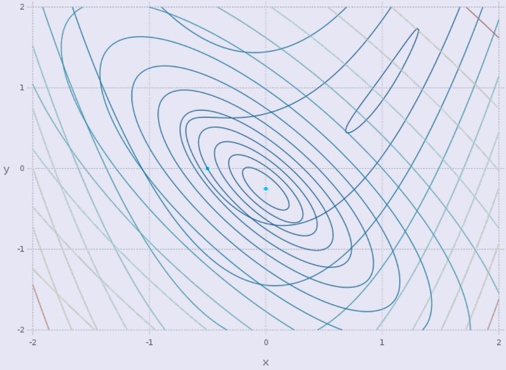
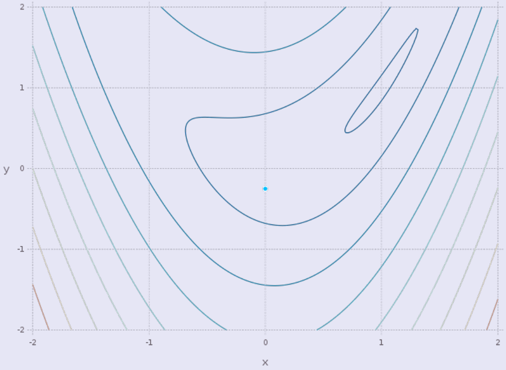


O Método de Newton para otimização

$$f(x) = (1 - x_1)^2 + 4(x_2 - x_1^2)^2, \quad x^1 = (0.000, -0.250)$$



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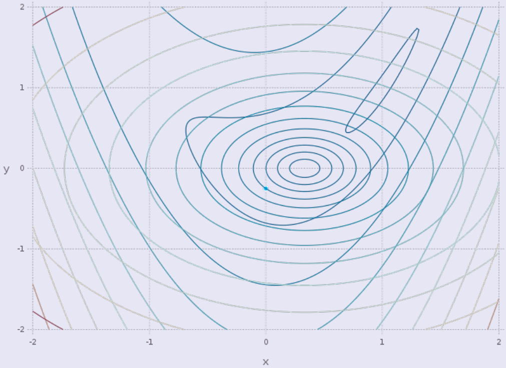

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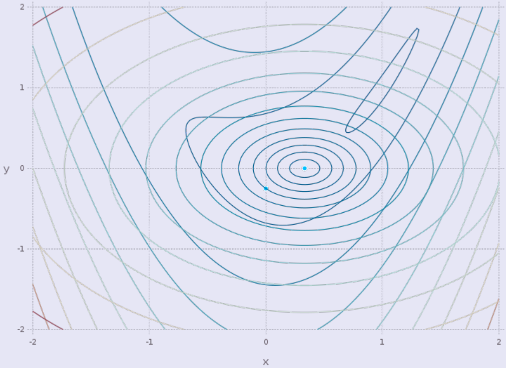
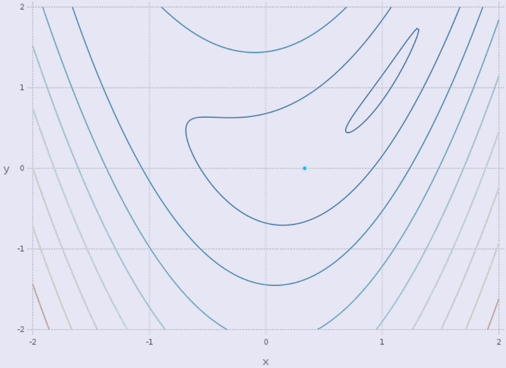


O Método de Newton para otimização

$$f(x) = (1 - x_1)^2 + 4(x_2 - x_1^2)^2, \quad x^2 = (0.333, 0.000)$$



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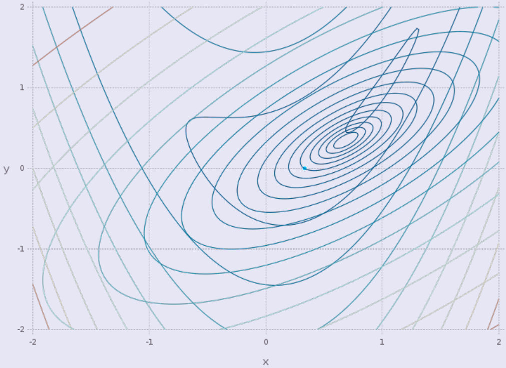

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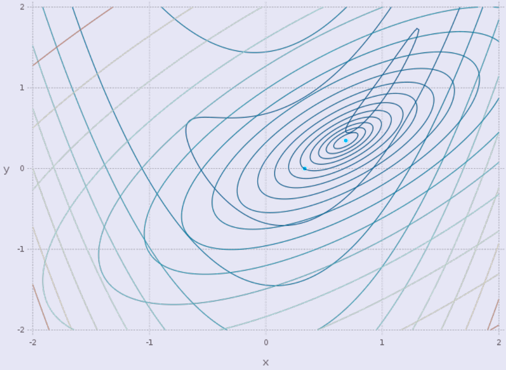
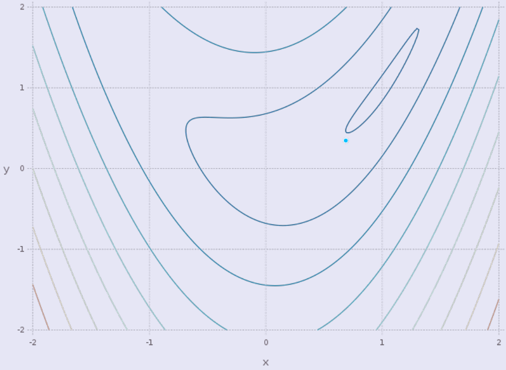


O Método de Newton para otimização

$$f(x) = (1 - x_1)^2 + 4(x_2 - x_1^2)^2, \quad x^3 = (0.686, 0.346)$$



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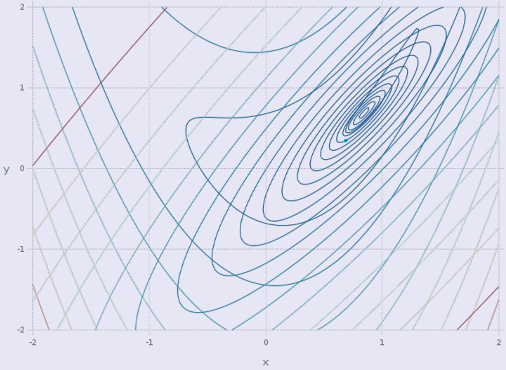

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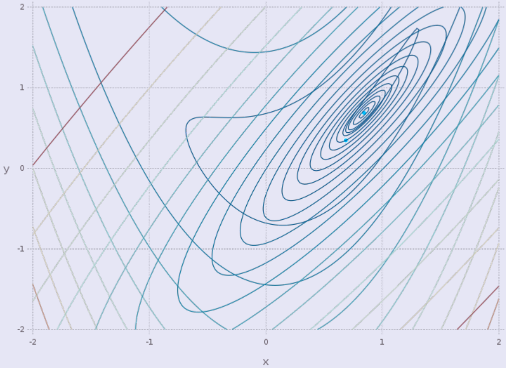
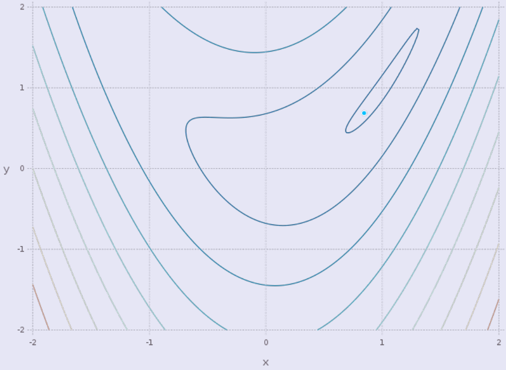


O Método de Newton para otimização

$$f(x) = (1 - x_1)^2 + 4(x_2 - x_1^2)^2, \quad x^4 = (0.843, 0.687)$$



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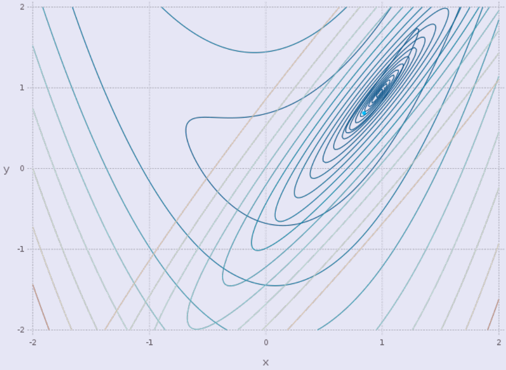

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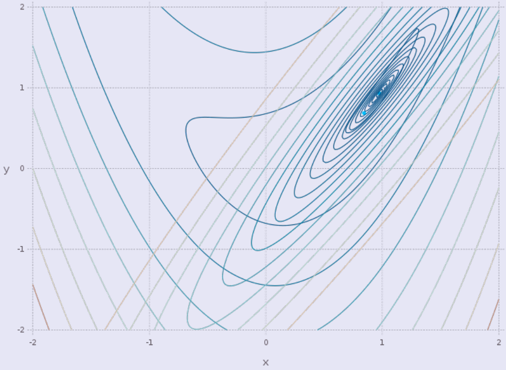

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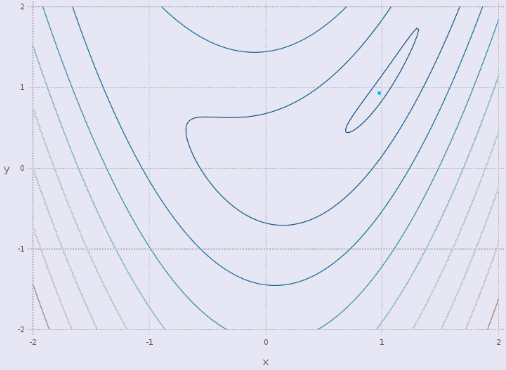

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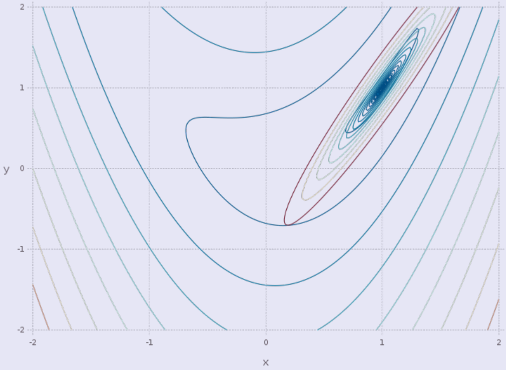

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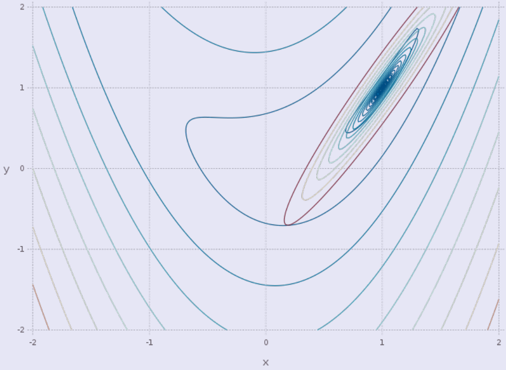

O Método de Newton para otimização

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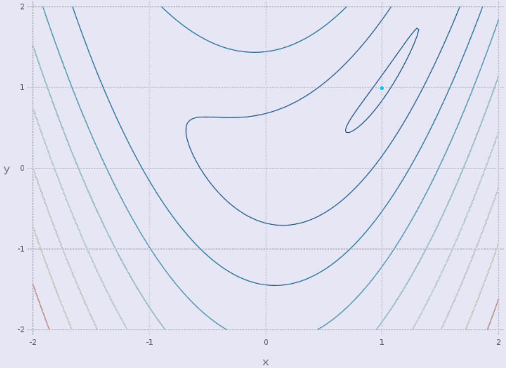

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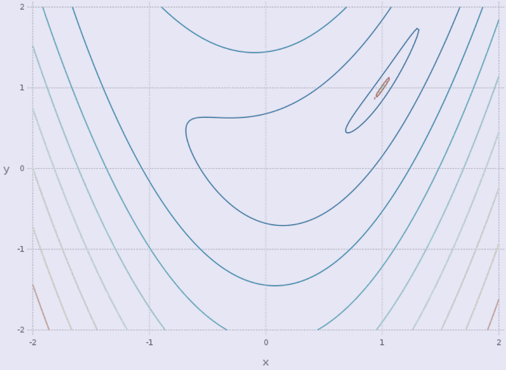
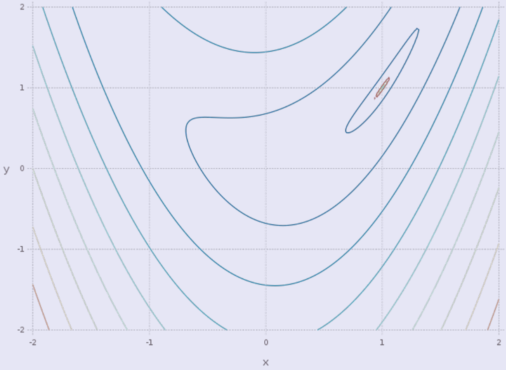


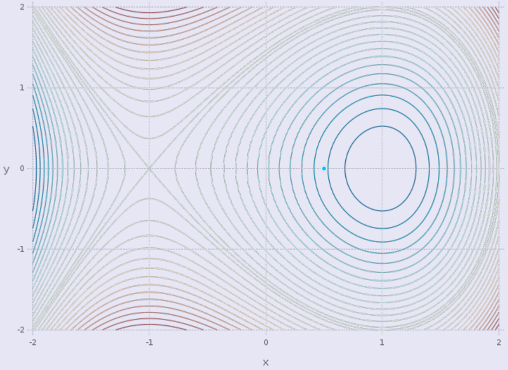
O Método de Newton para otimização

$$f(x) = (1 - x_1)^2 + 4(x_2 - x_1^2)^2, \quad x^6 = (0.997, 0.993)$$



O Método de Newton para otimização

$$f(x) = x_1^3 - 3x_1 + x_2^2$$



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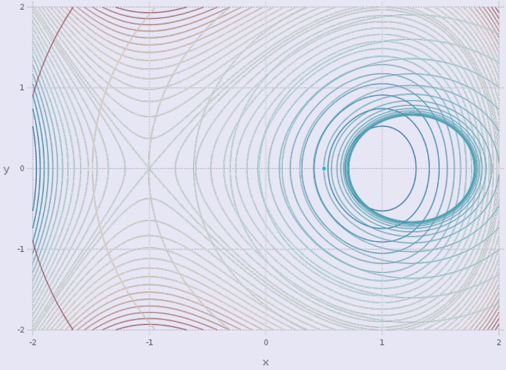

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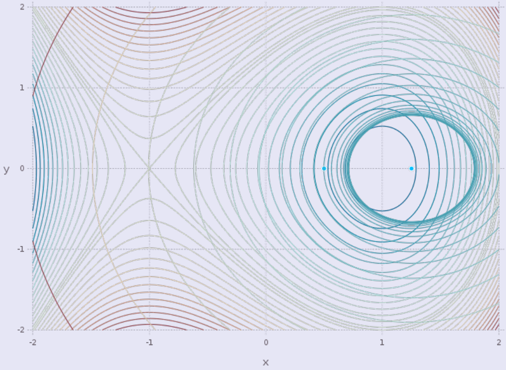

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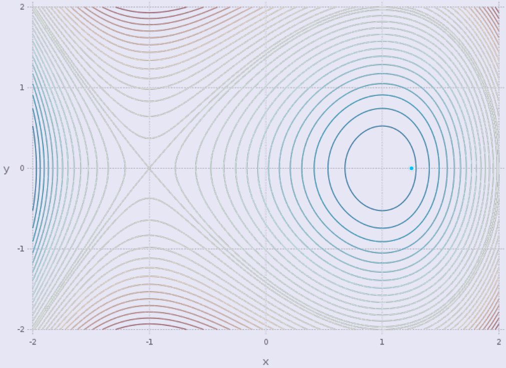

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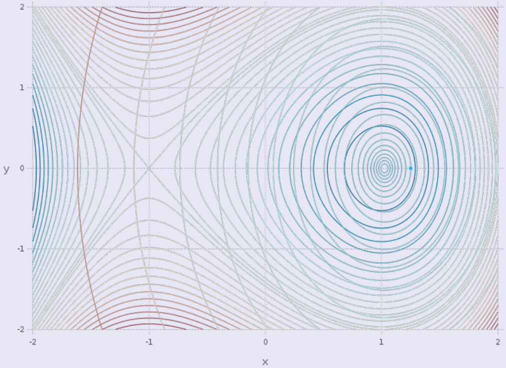

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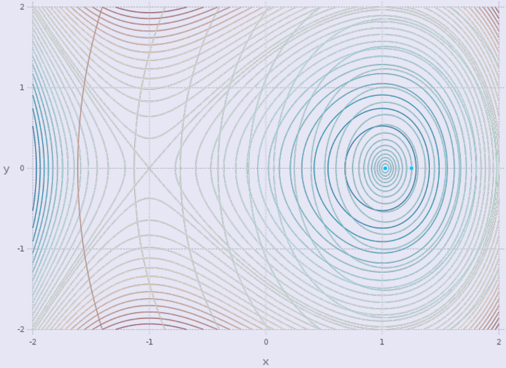

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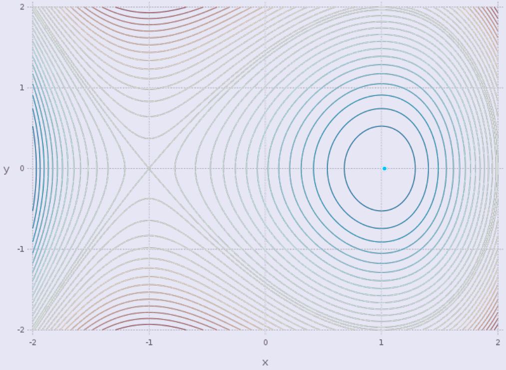

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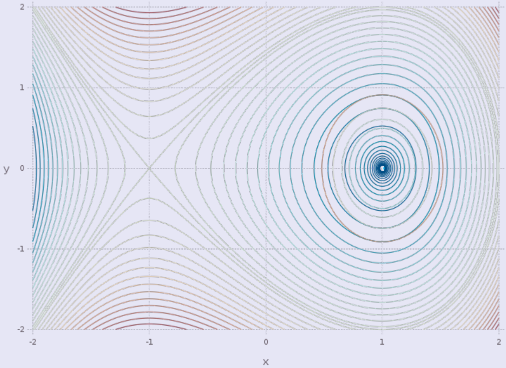

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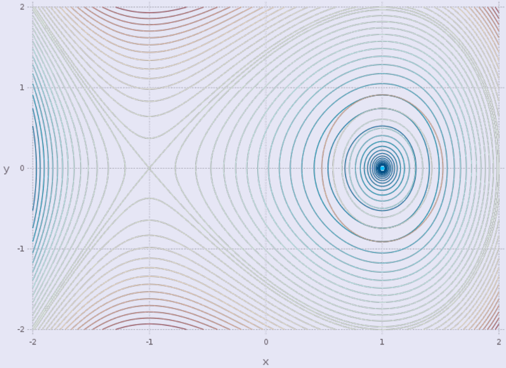

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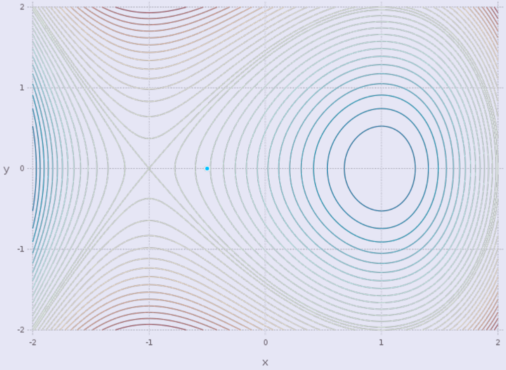

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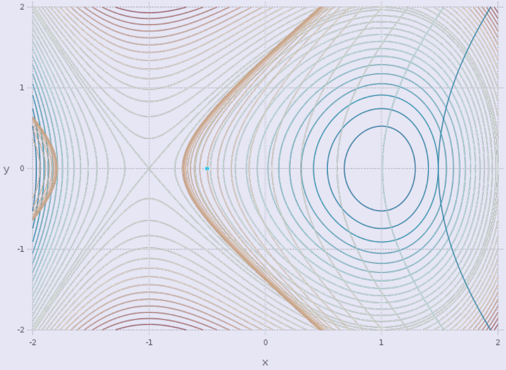

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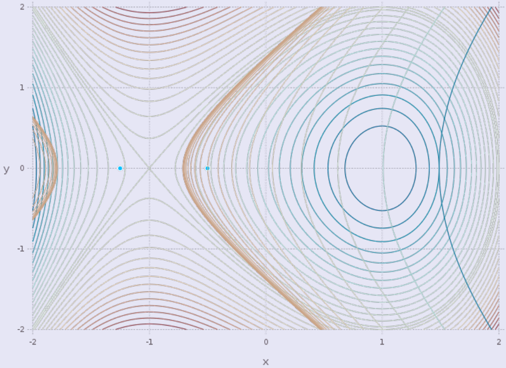

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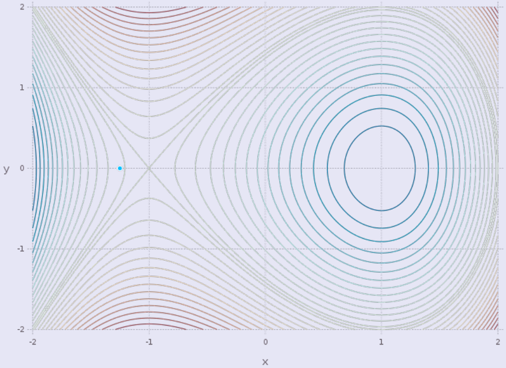

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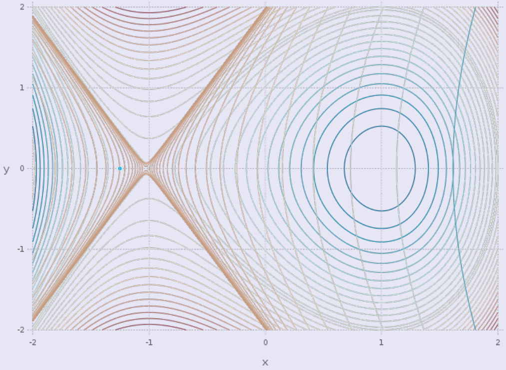

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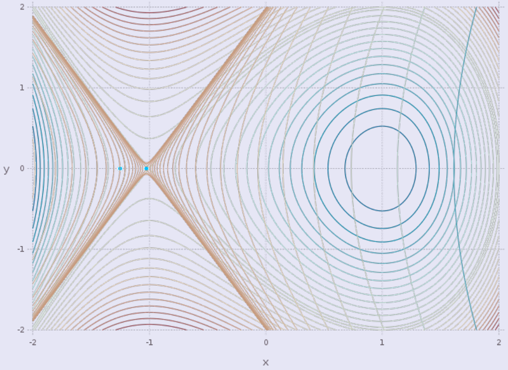

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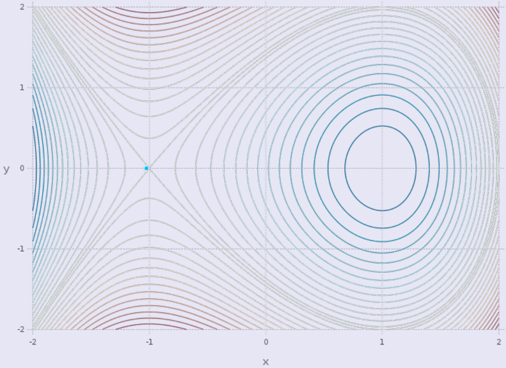

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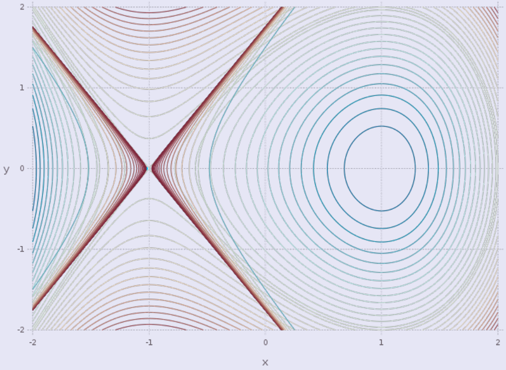

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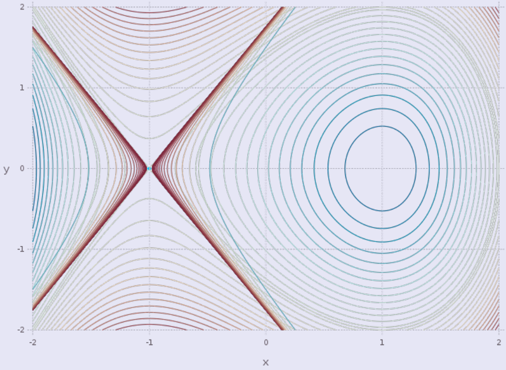

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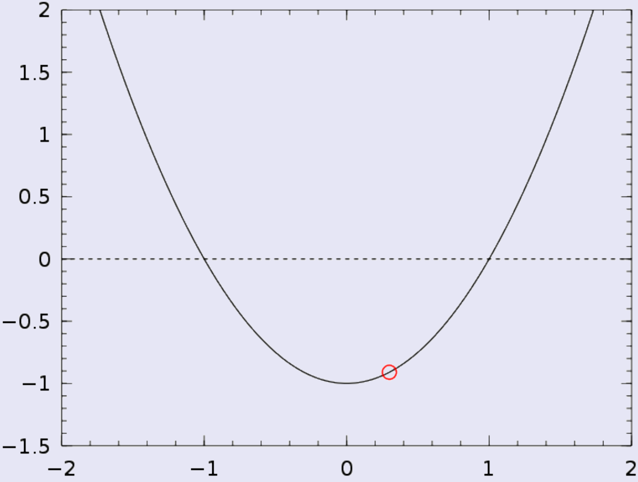

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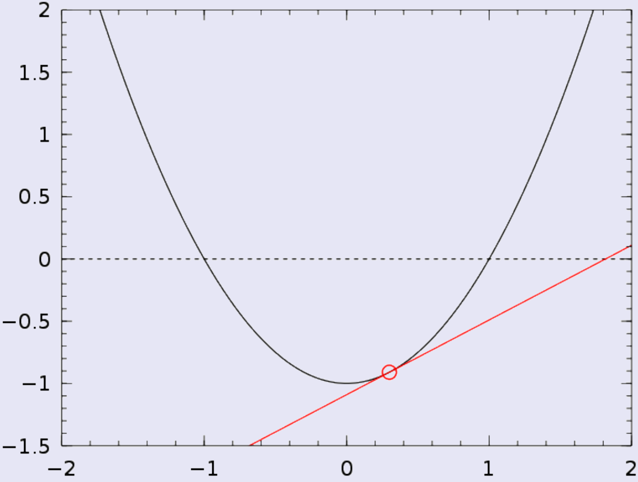

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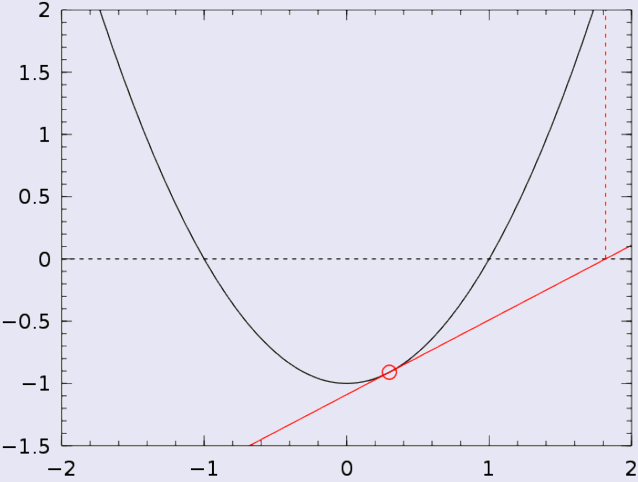

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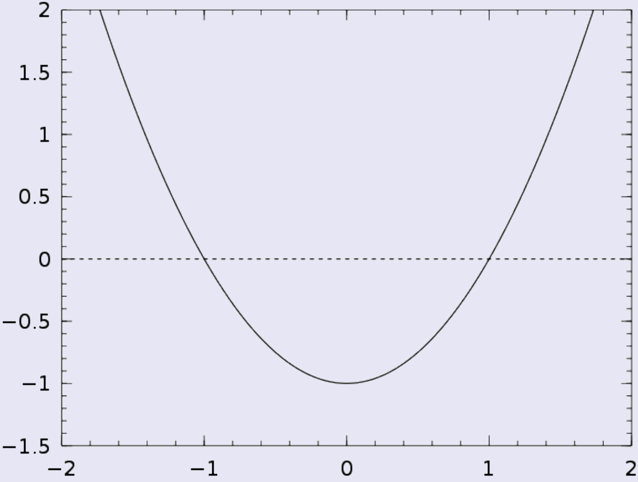

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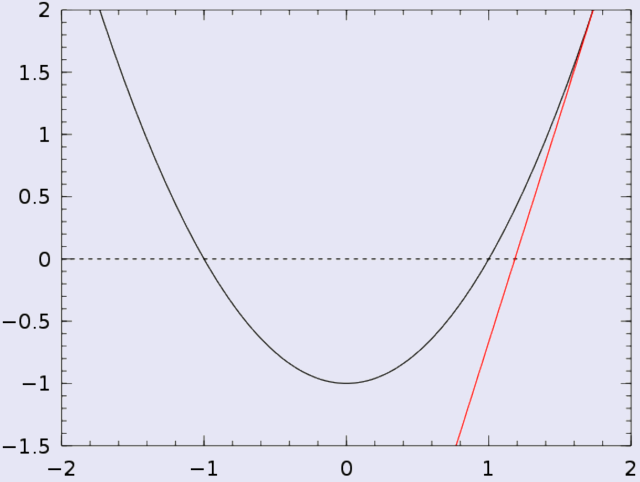

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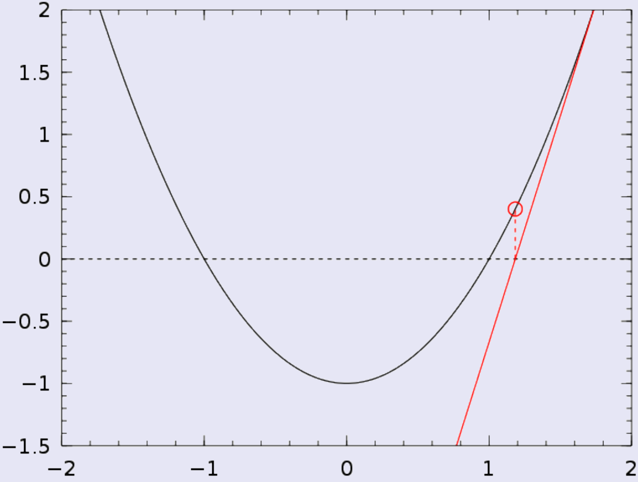

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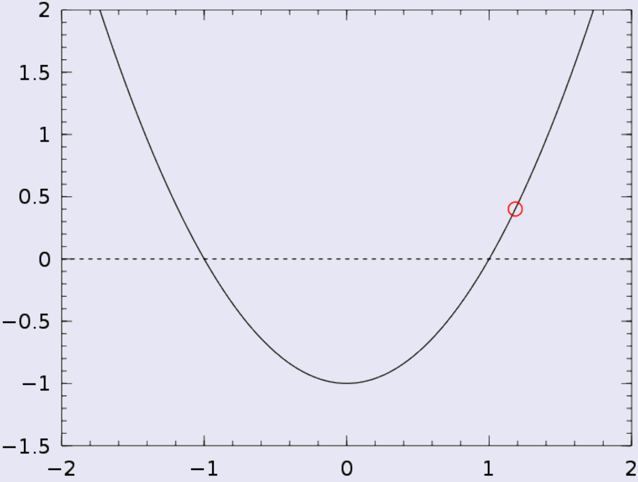

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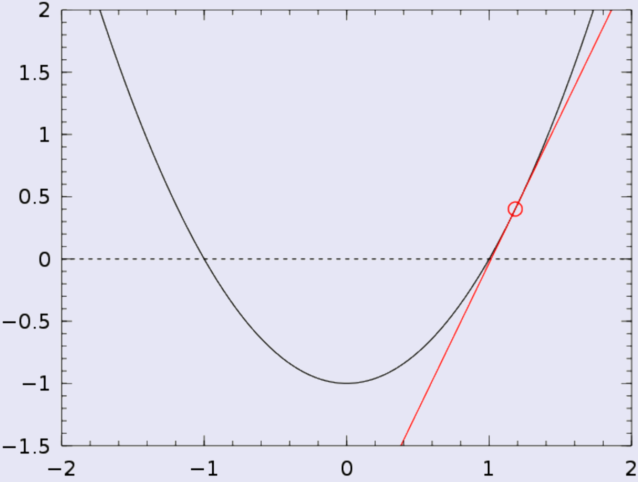

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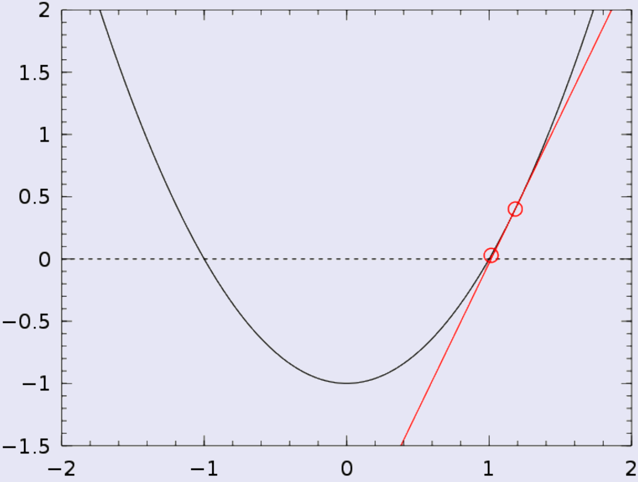

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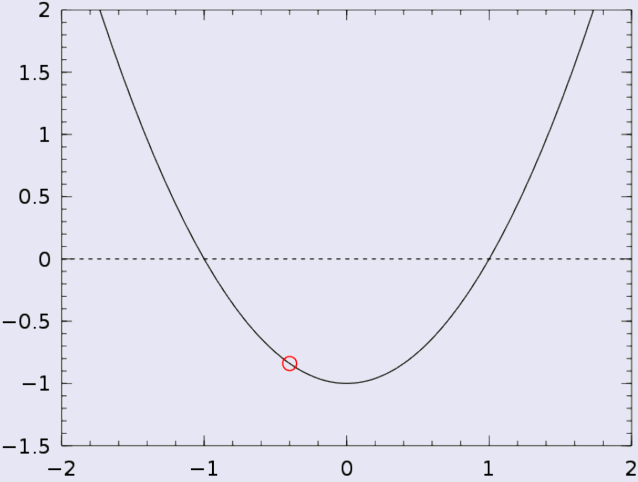

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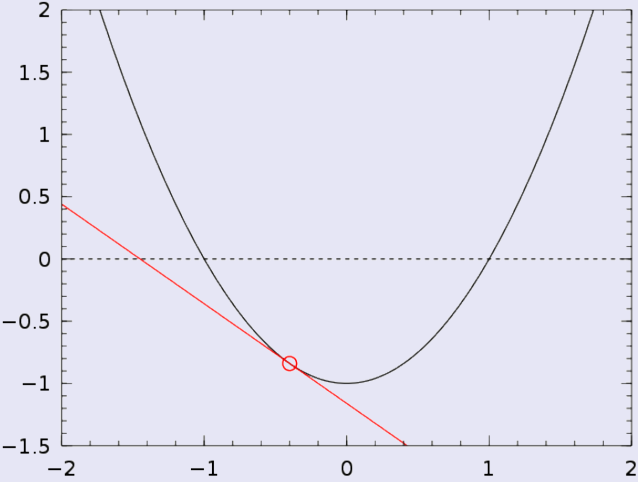

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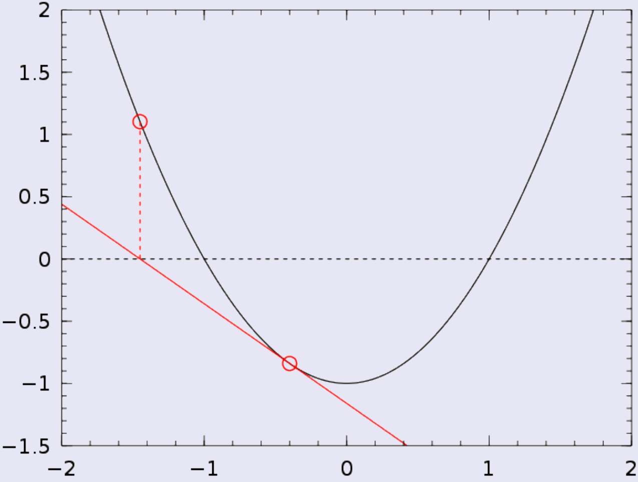

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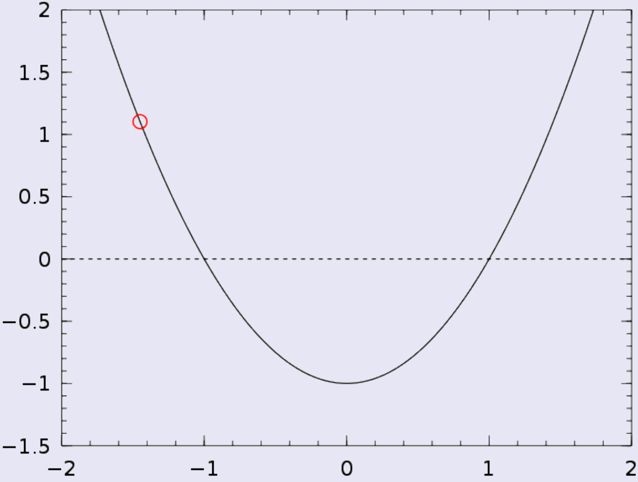

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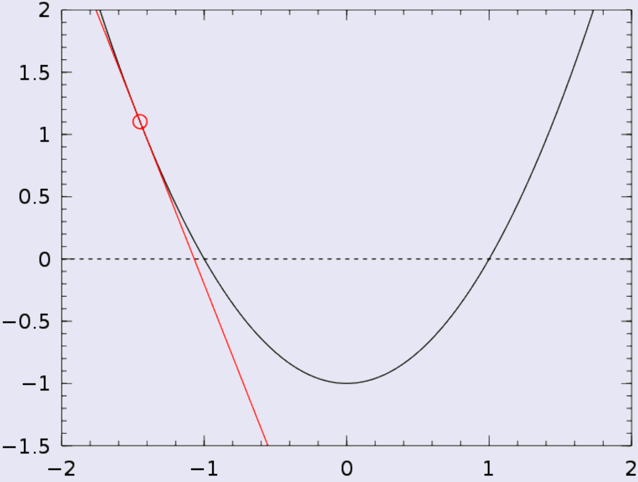

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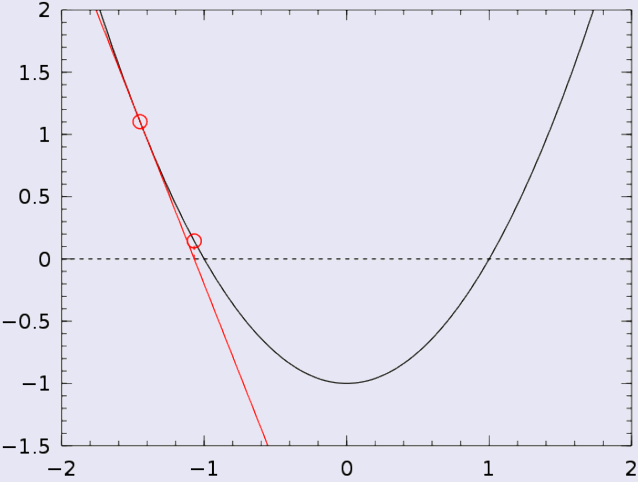

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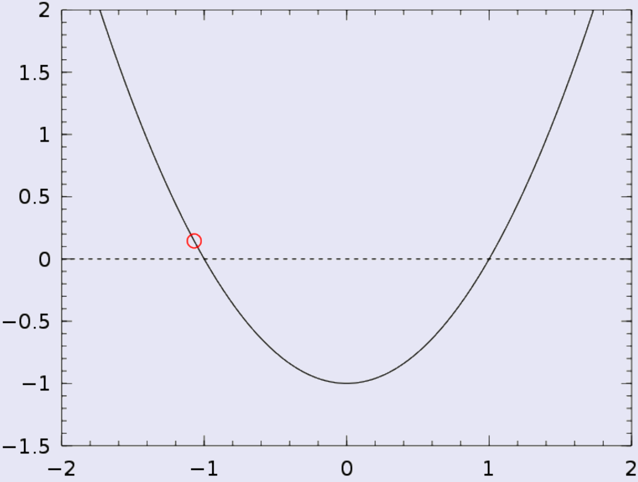

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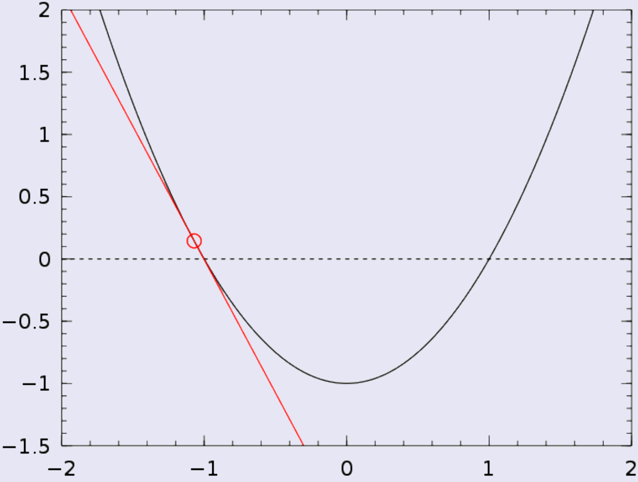

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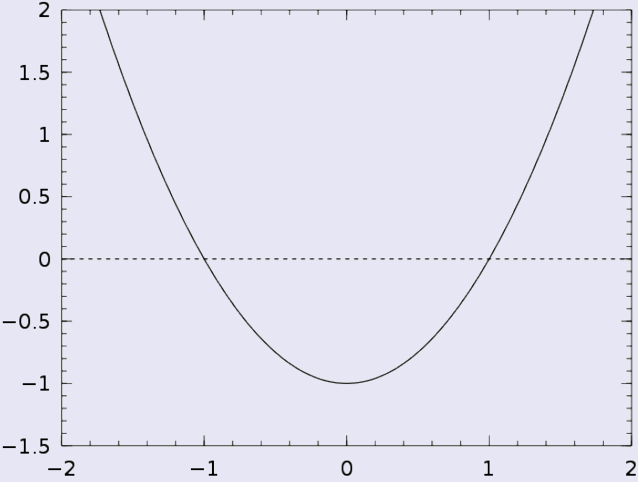

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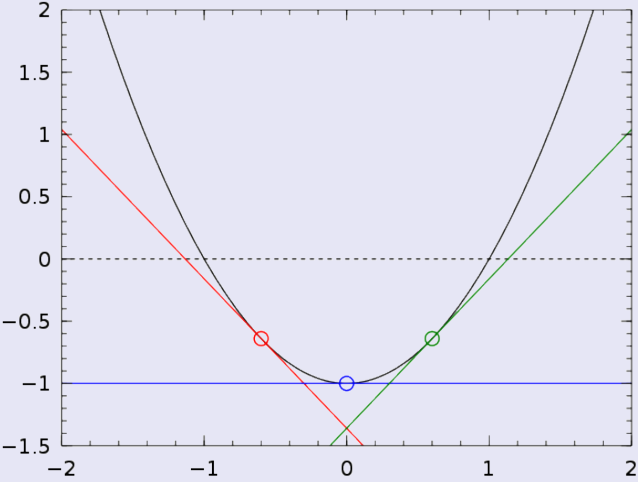

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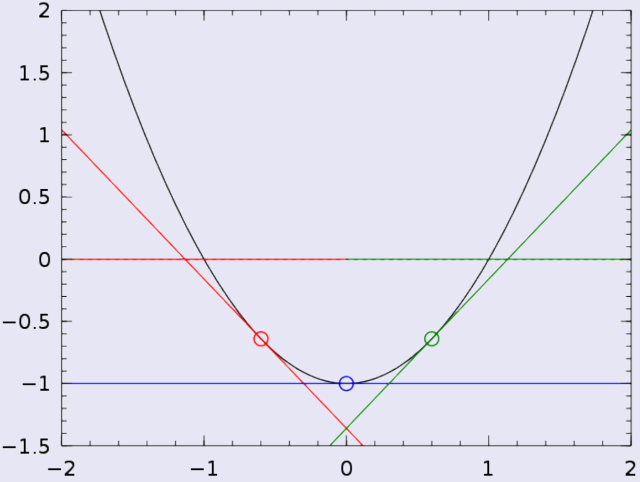

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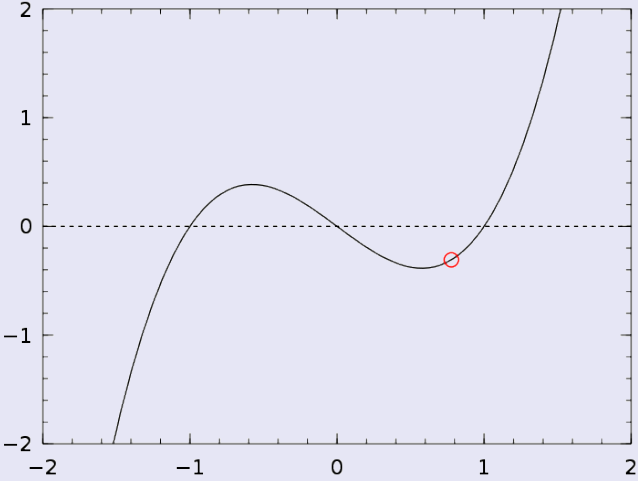

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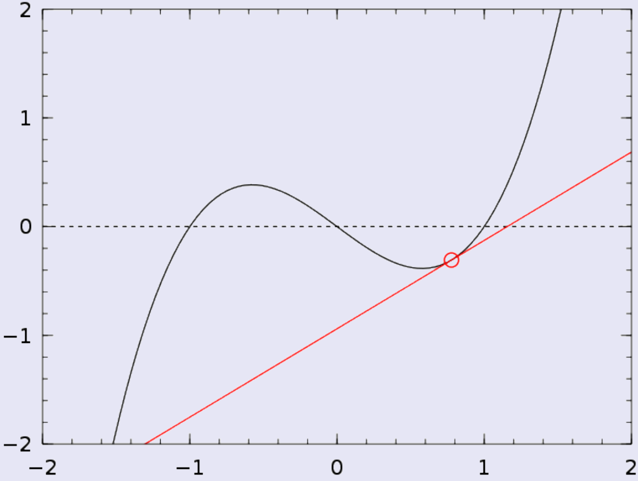

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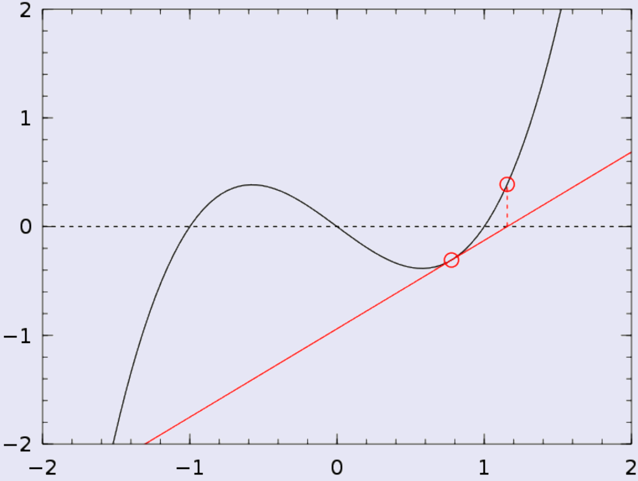

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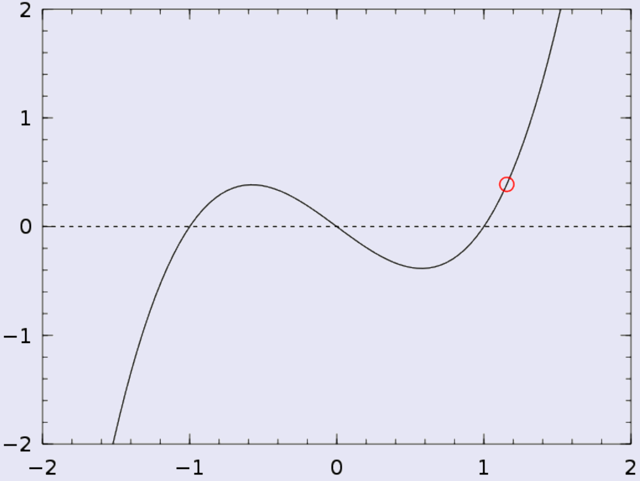

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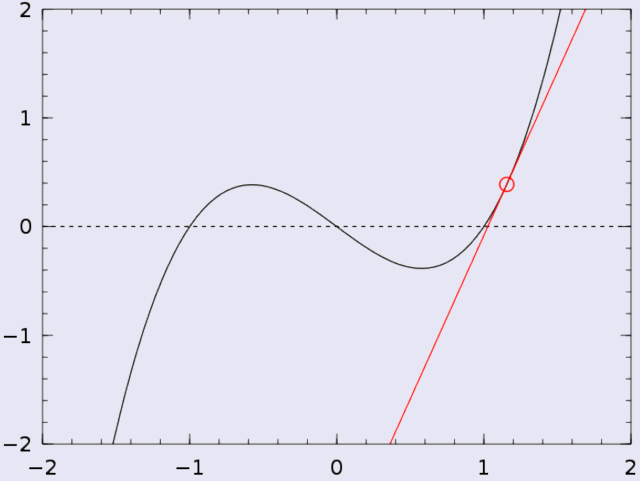

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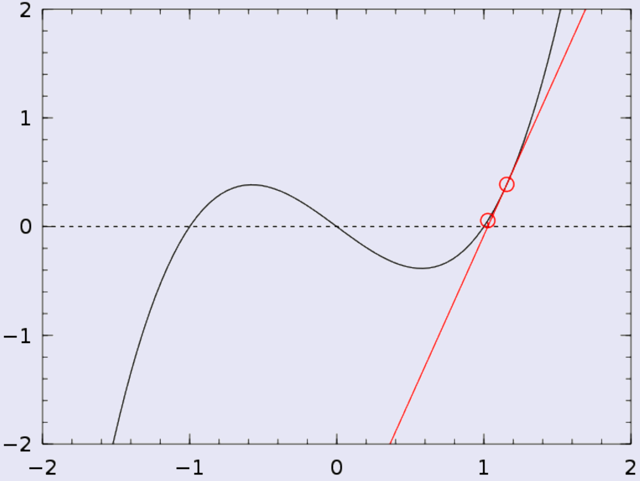

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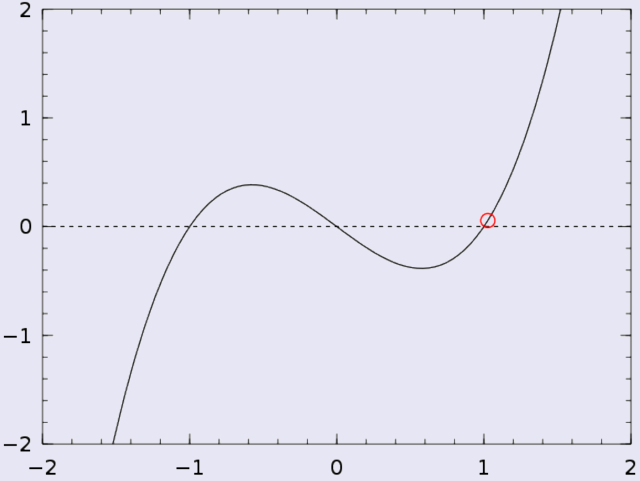

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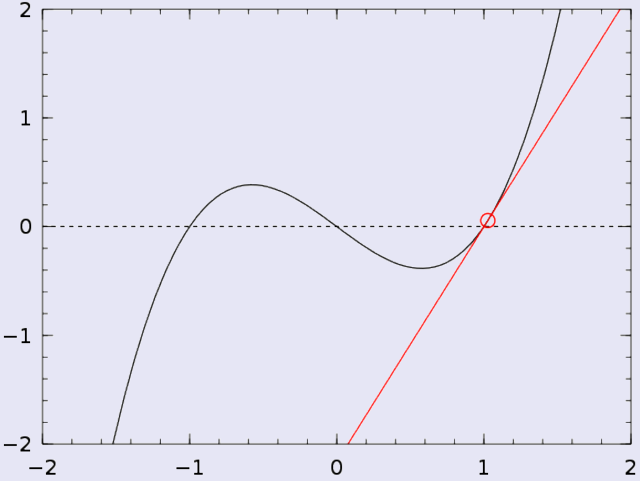

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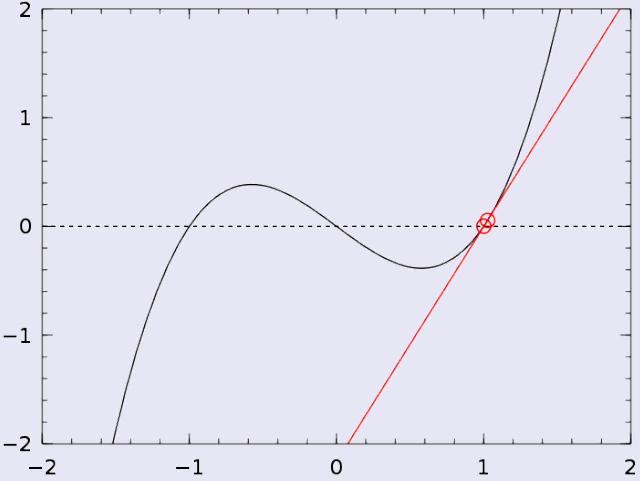

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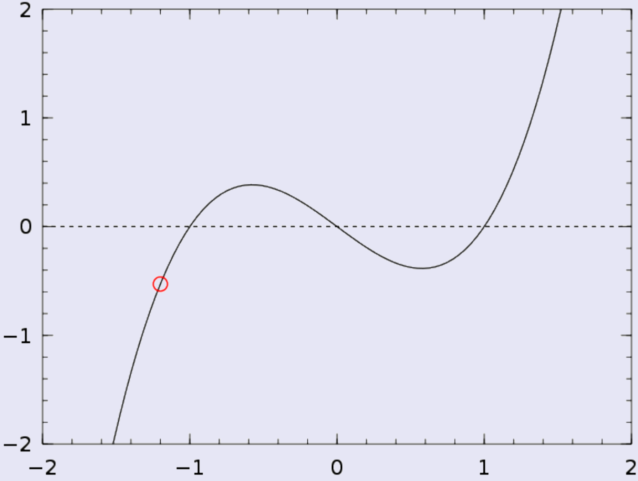

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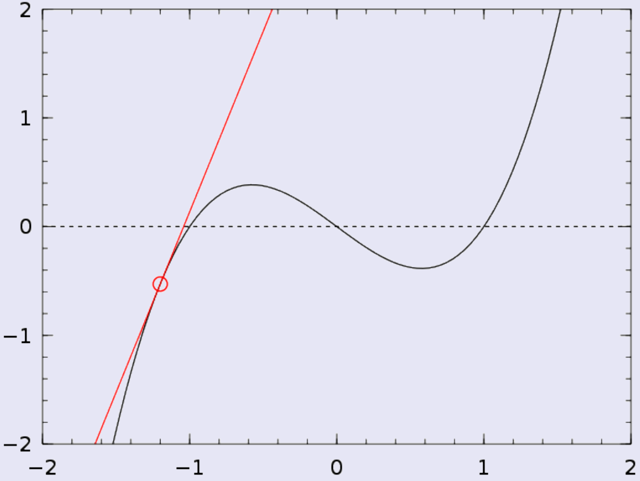

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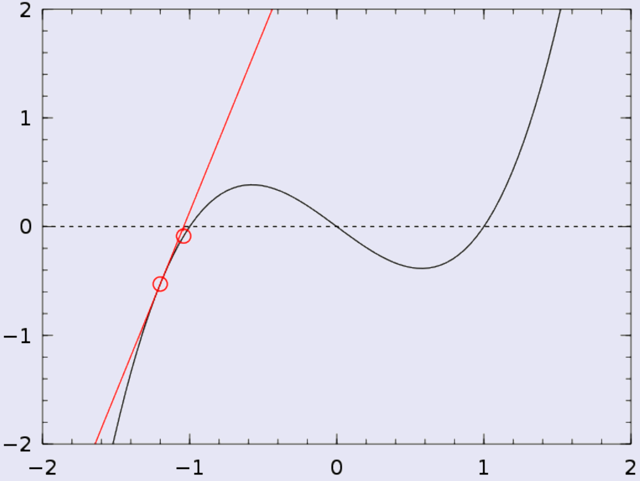

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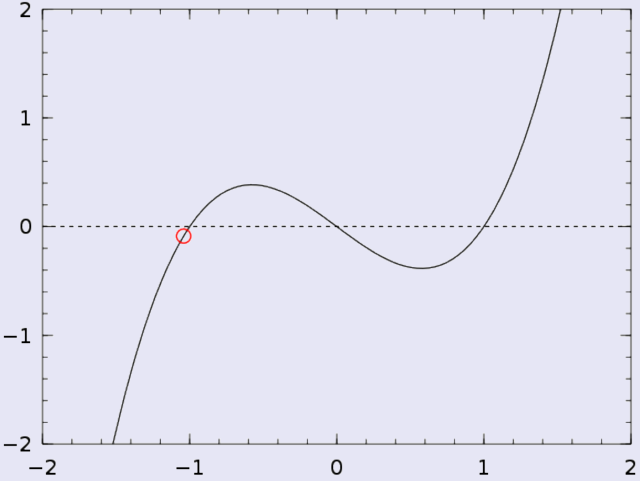

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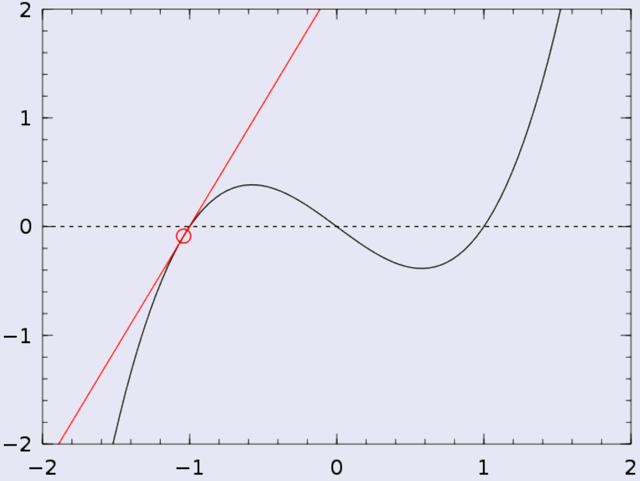

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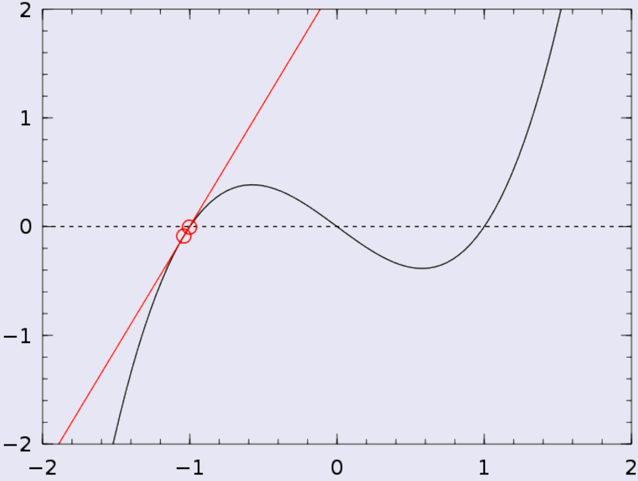

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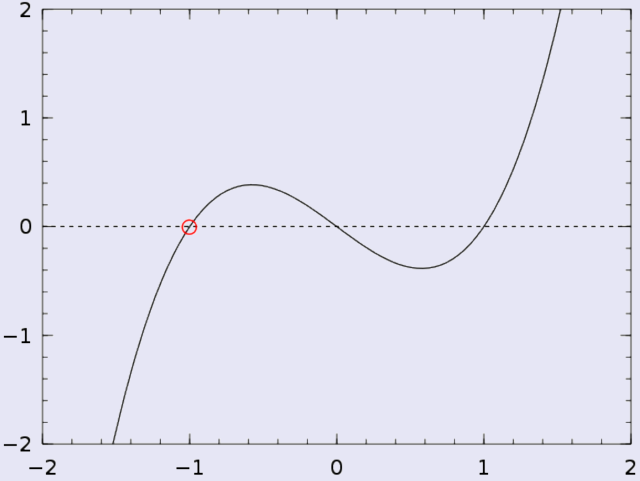

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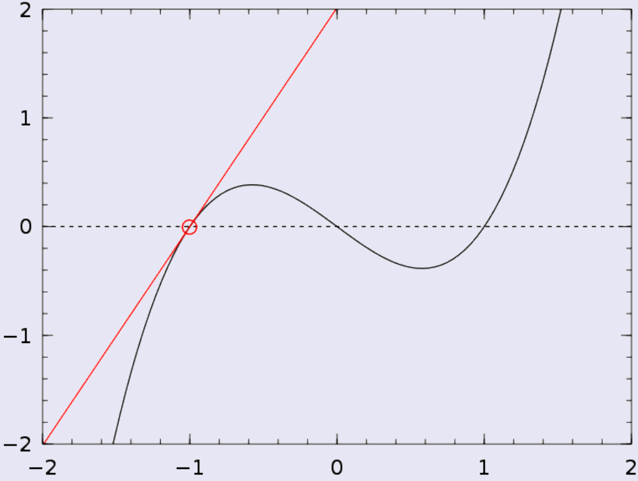

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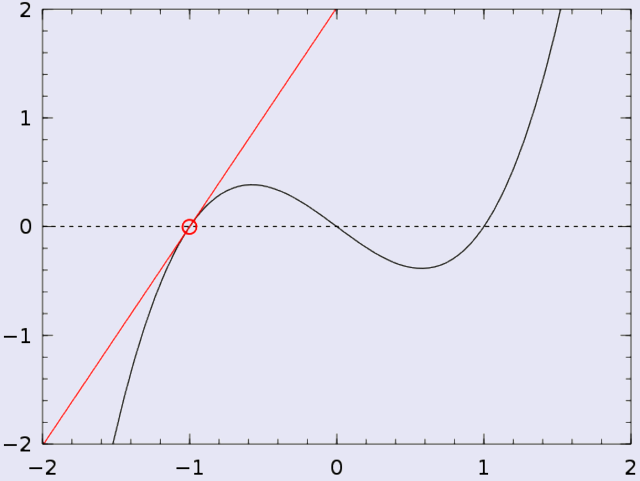

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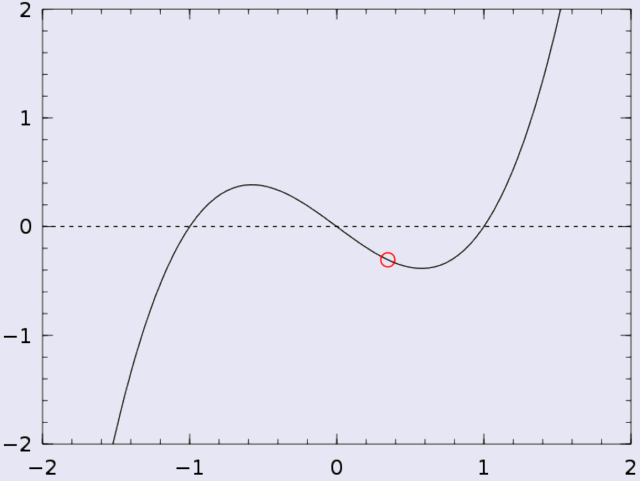

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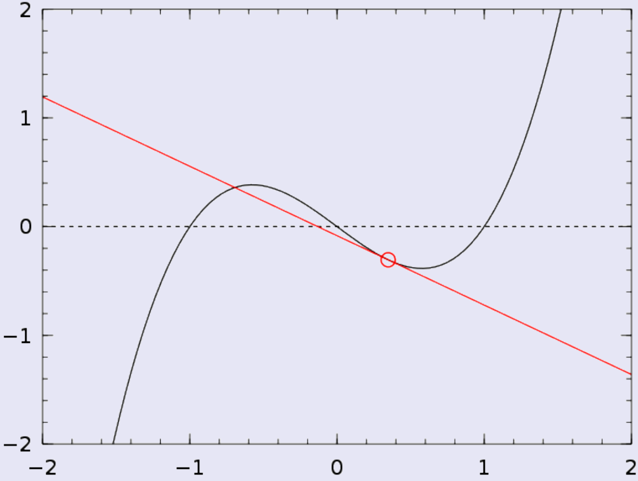

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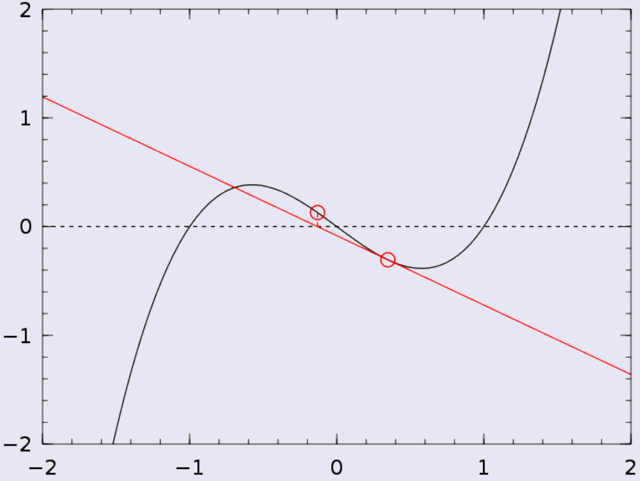

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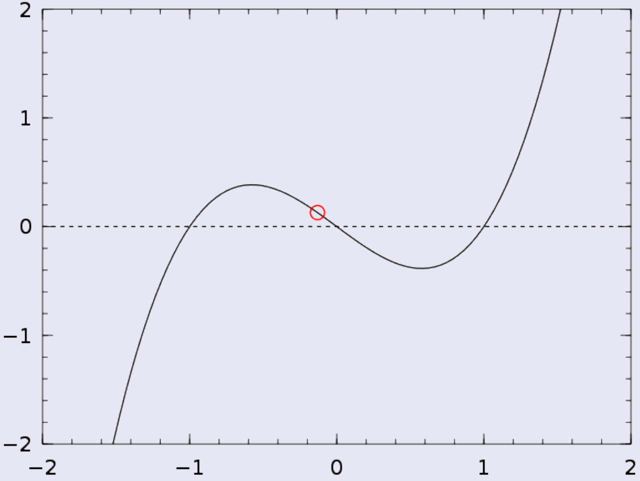

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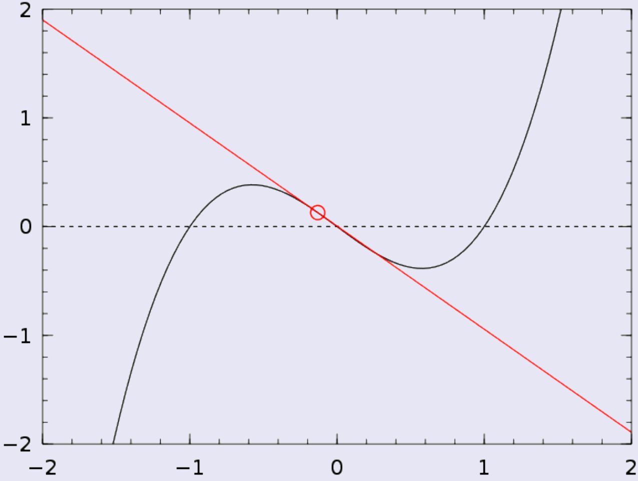

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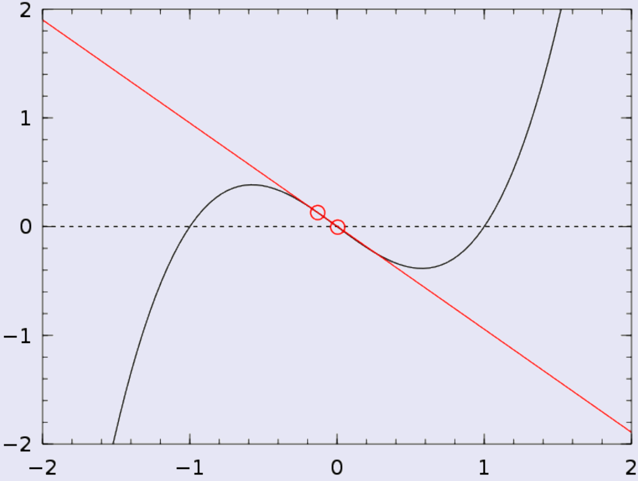

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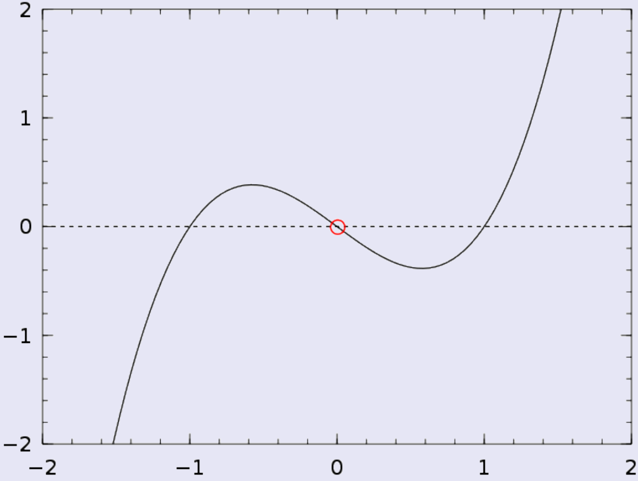

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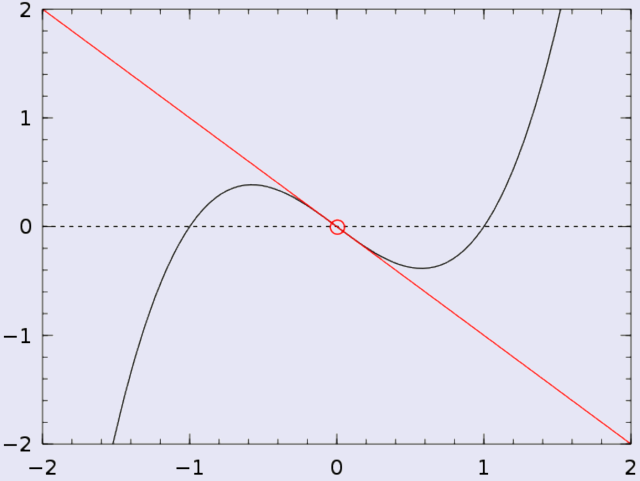

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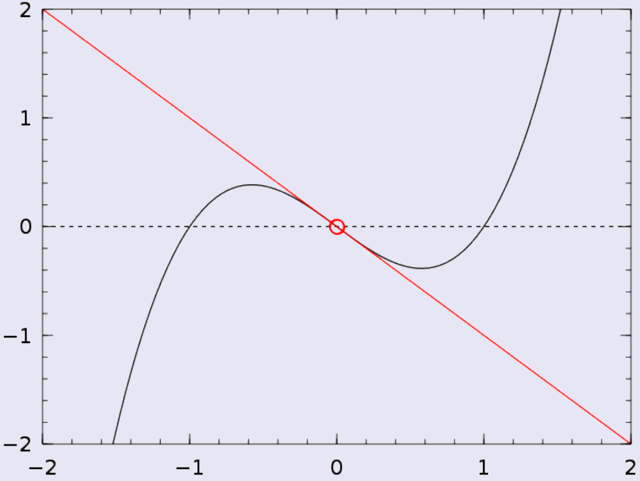

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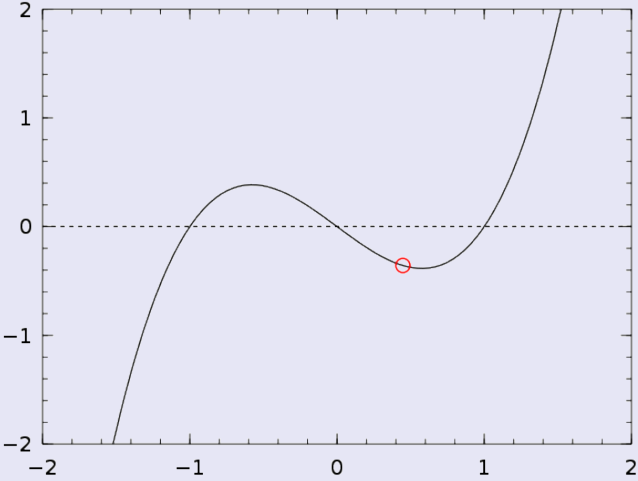

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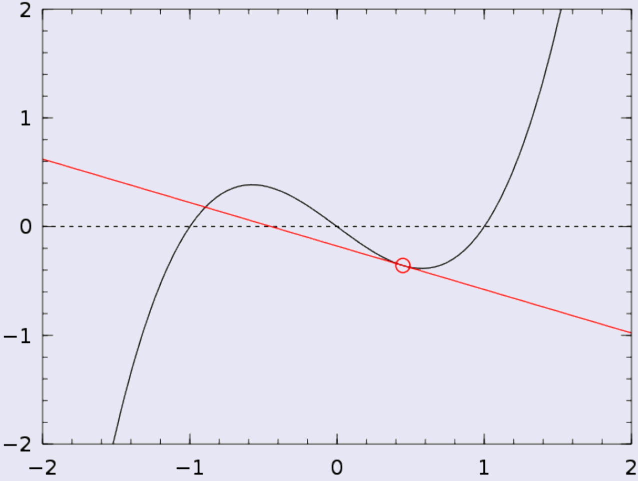

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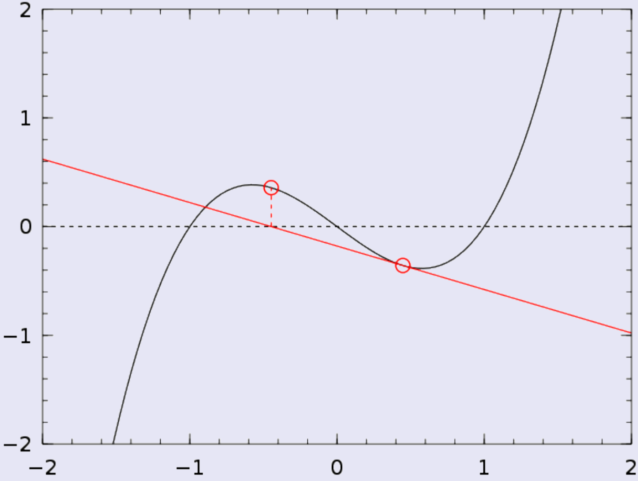

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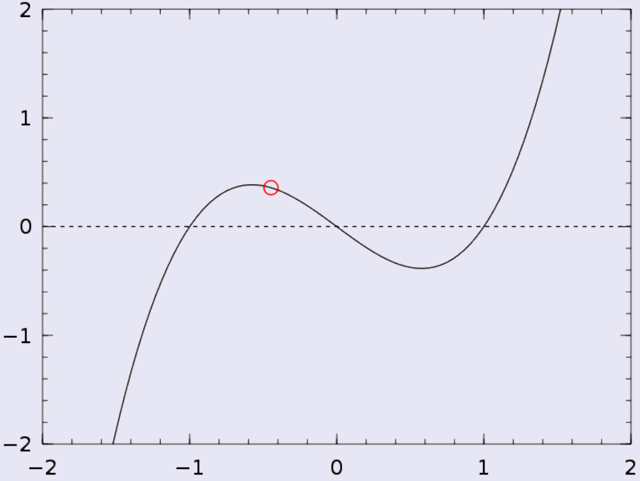

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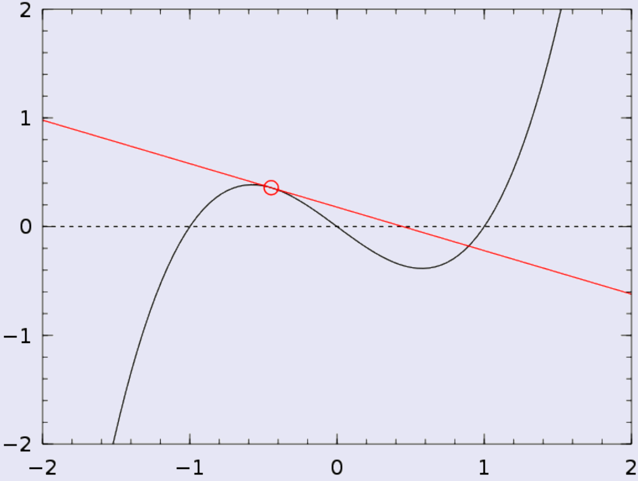

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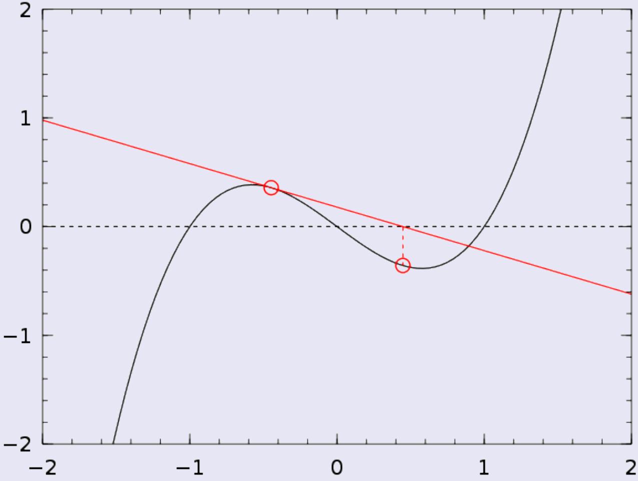

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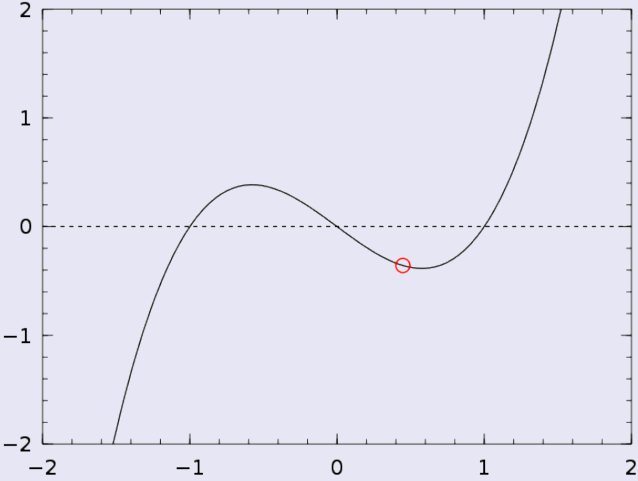

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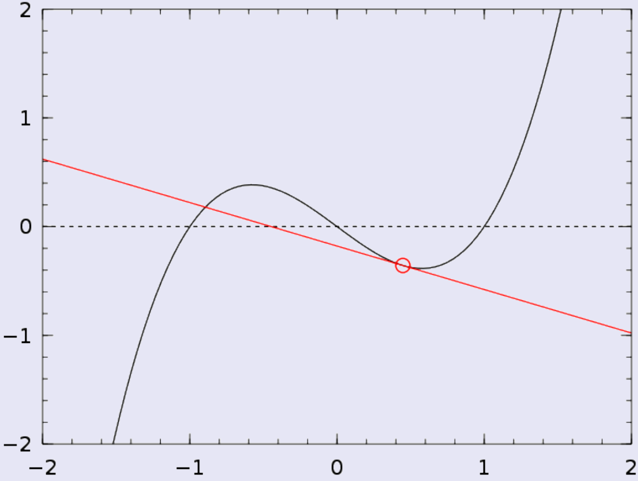

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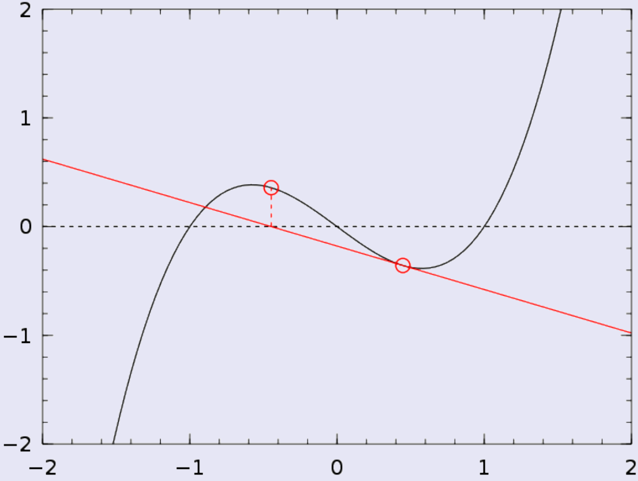

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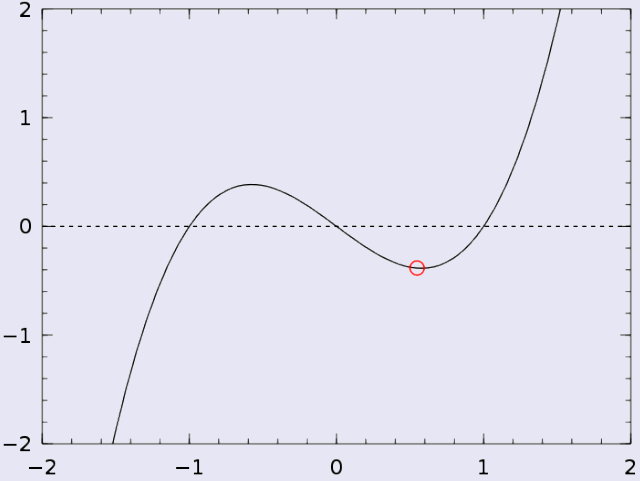

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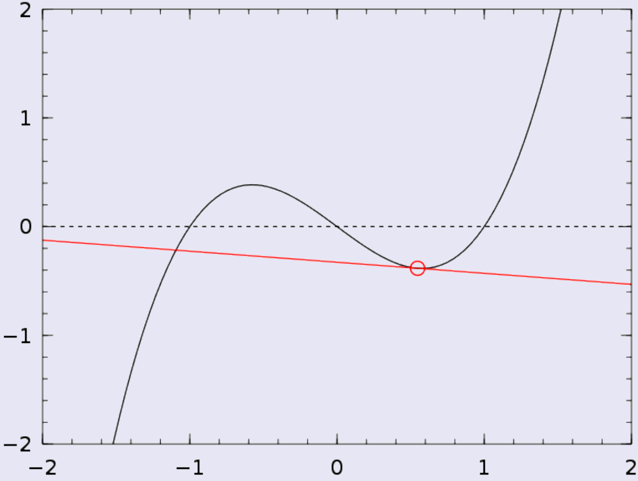

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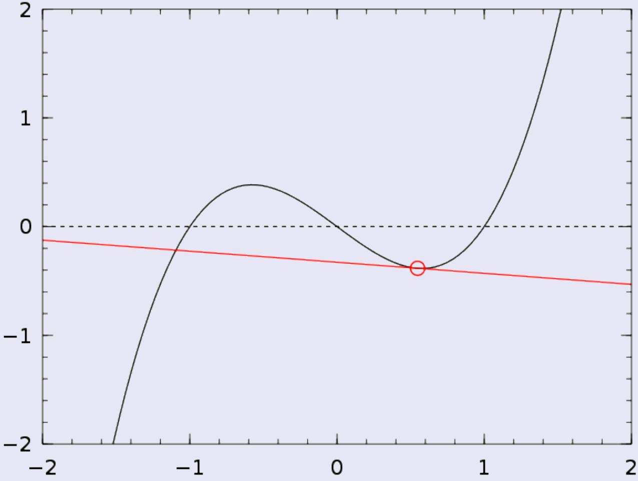

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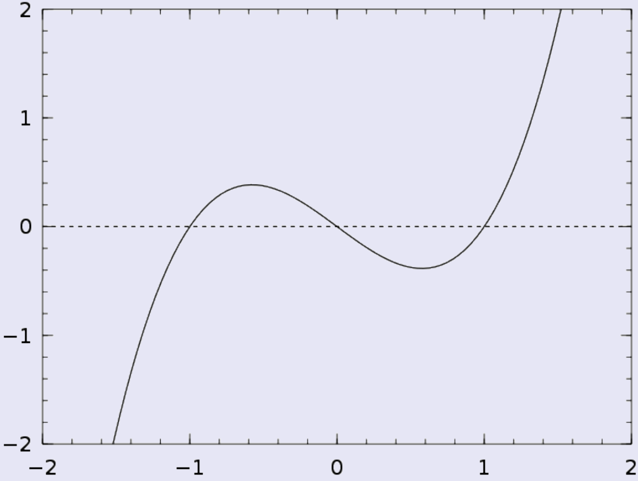

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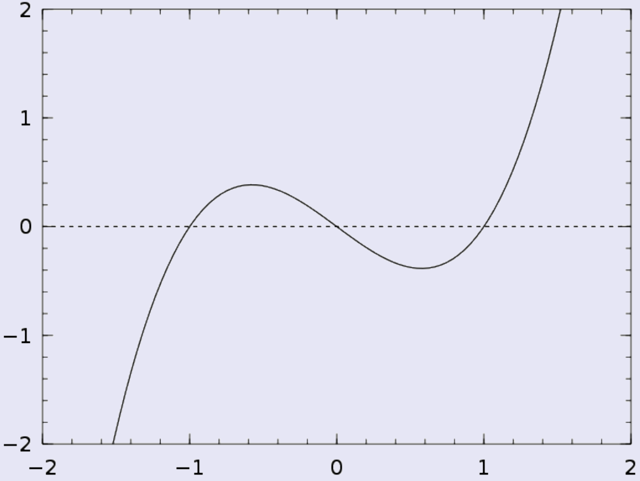

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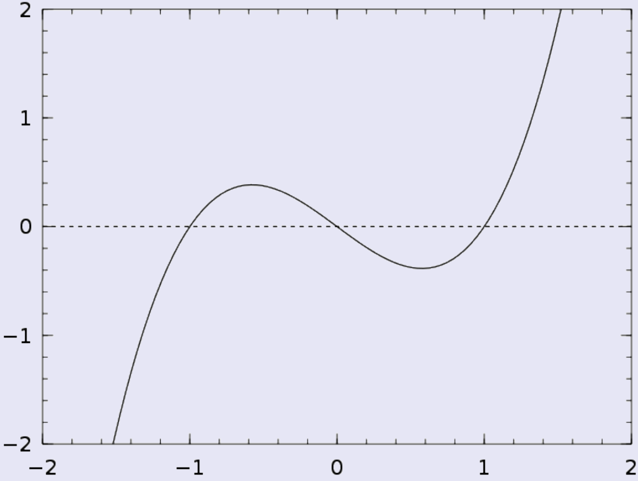

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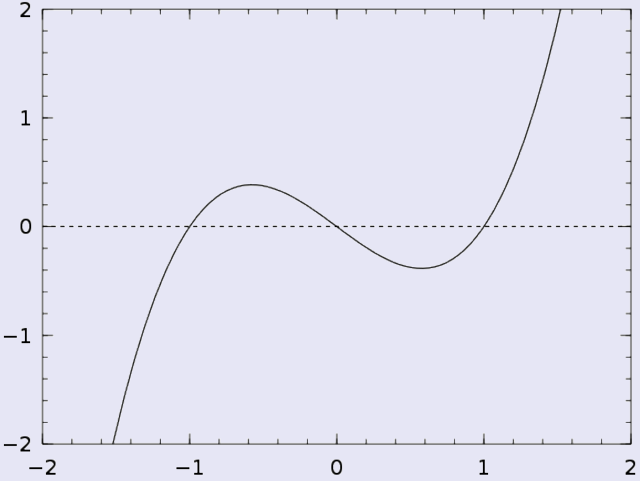

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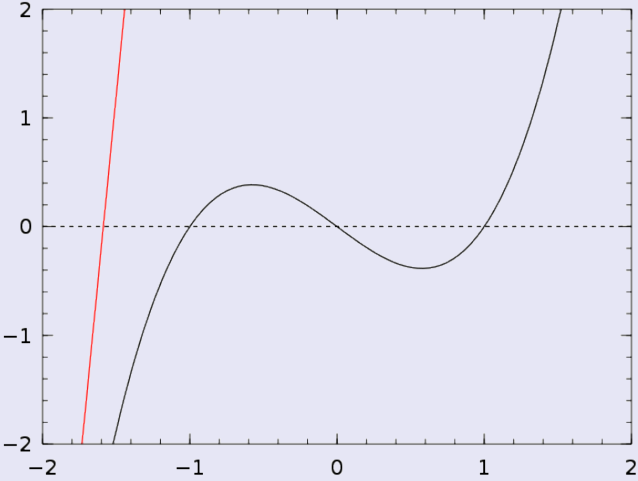

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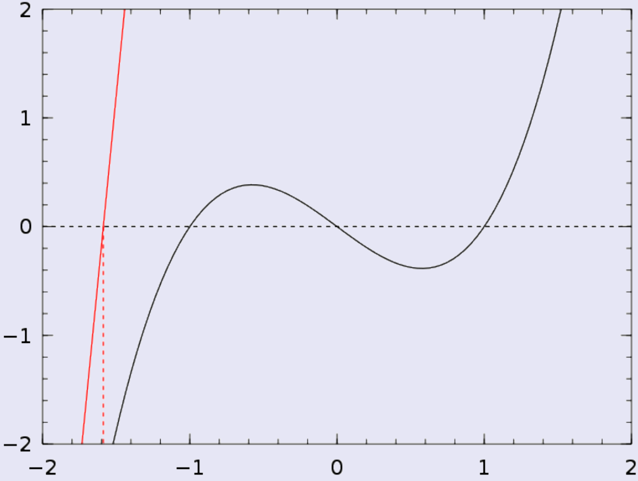

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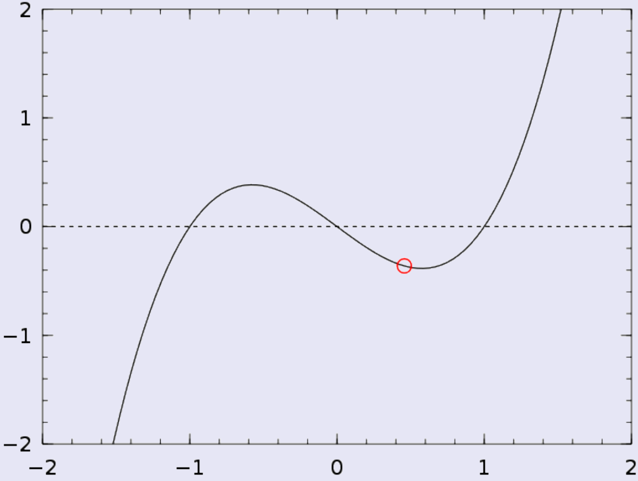

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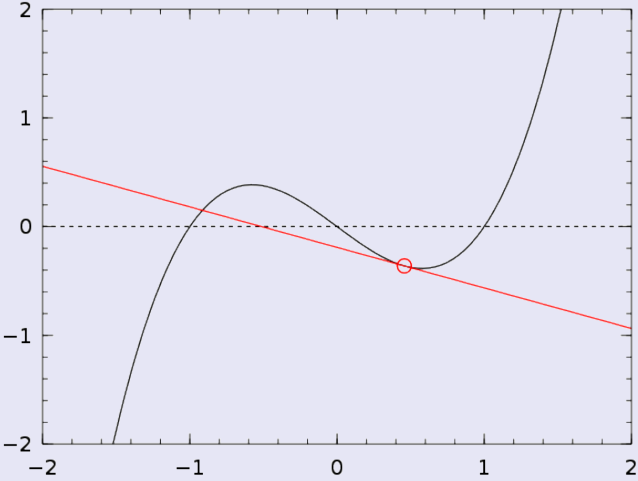

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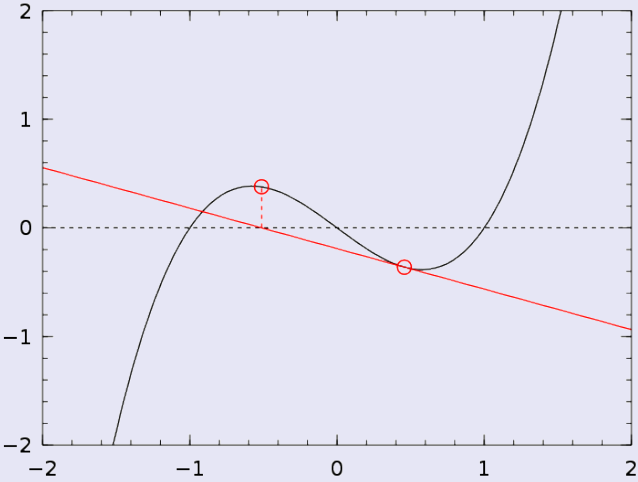

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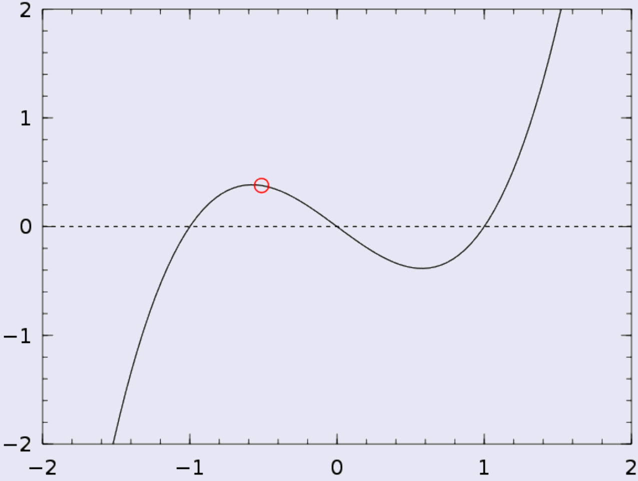

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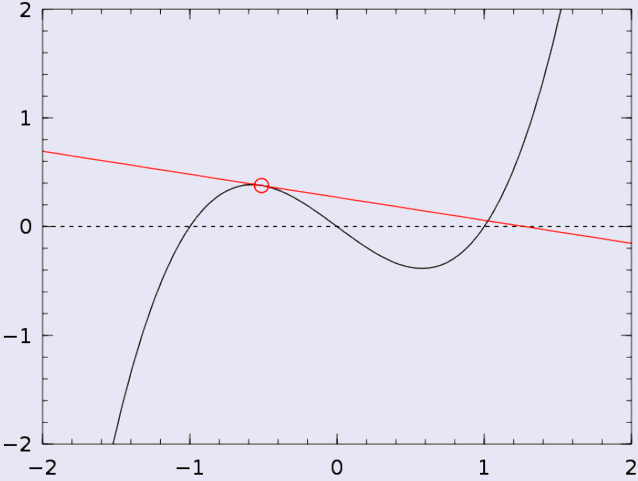

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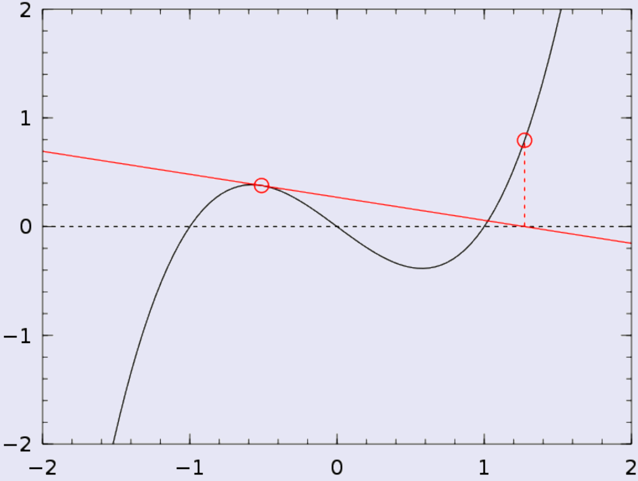

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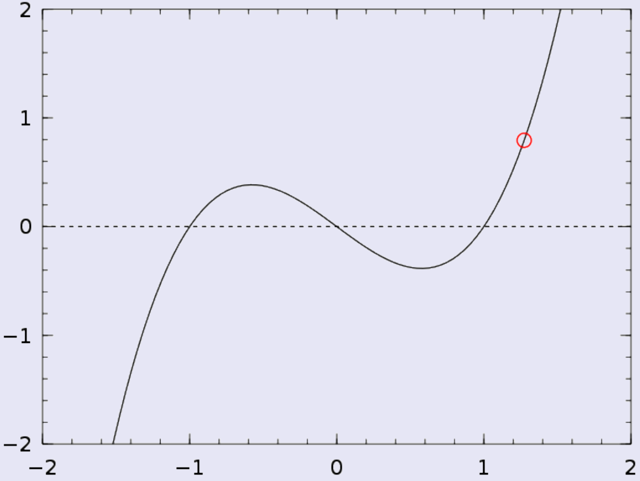

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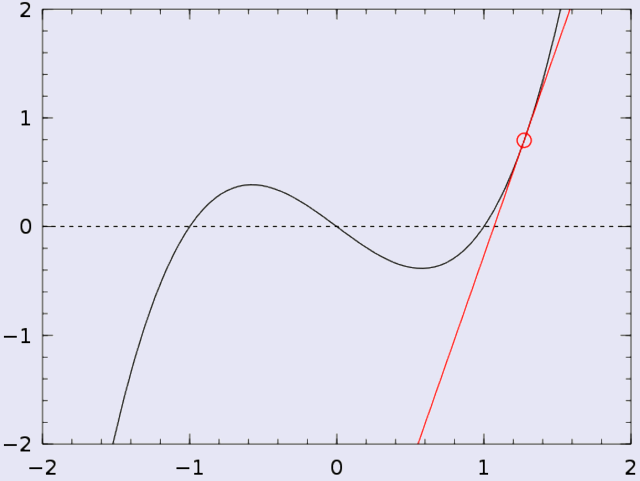

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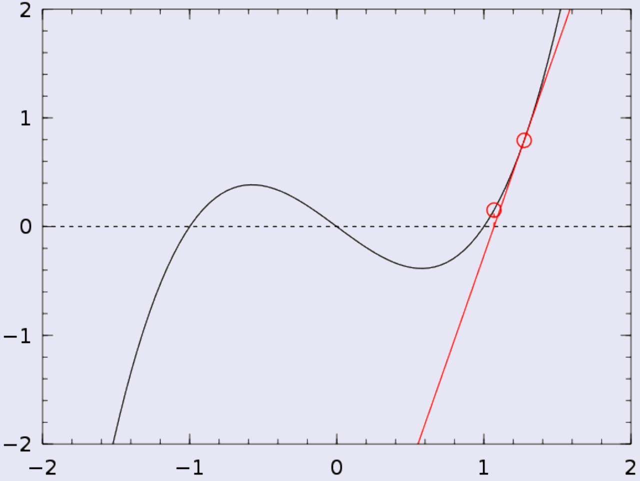

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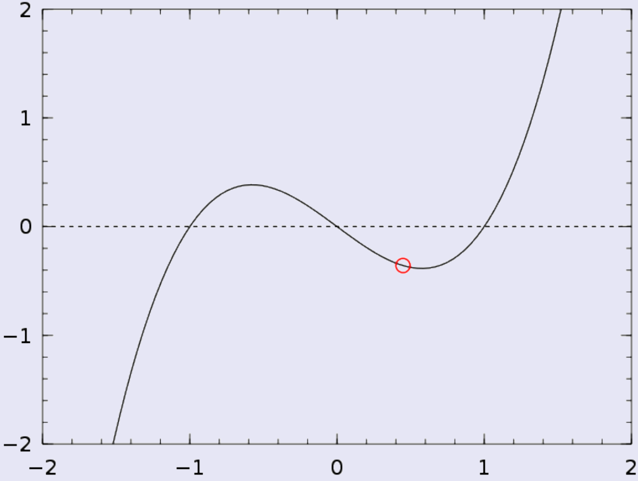

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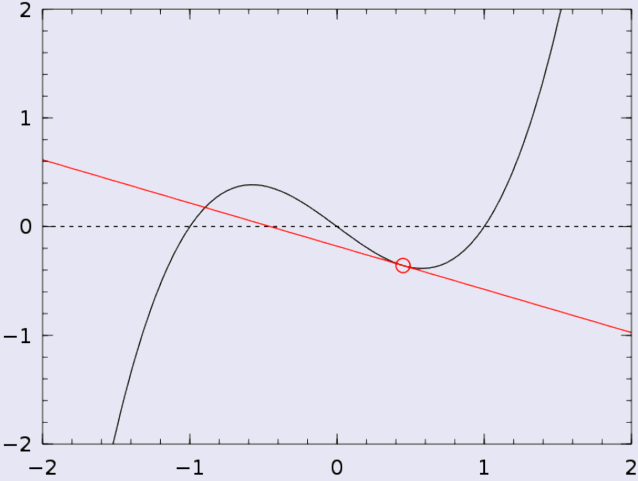

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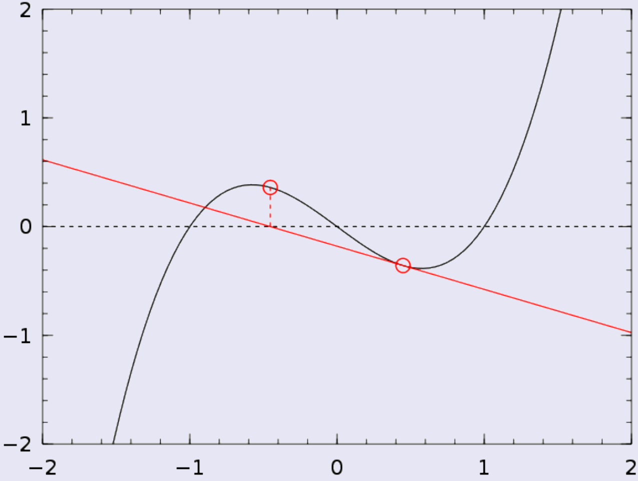

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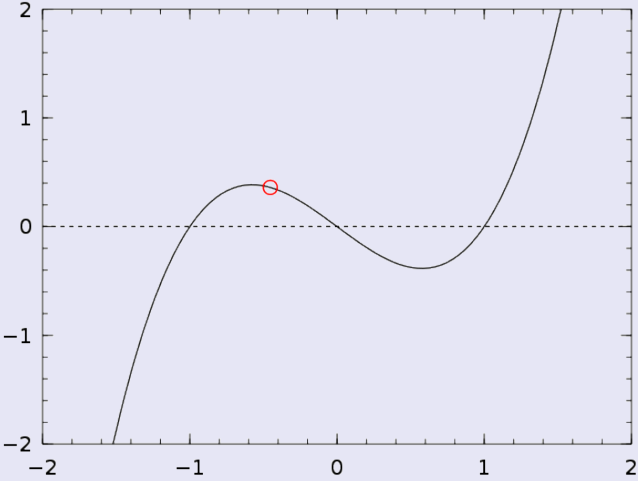

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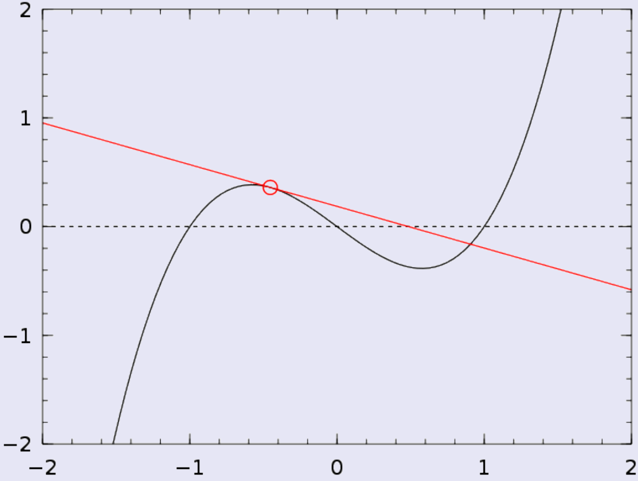

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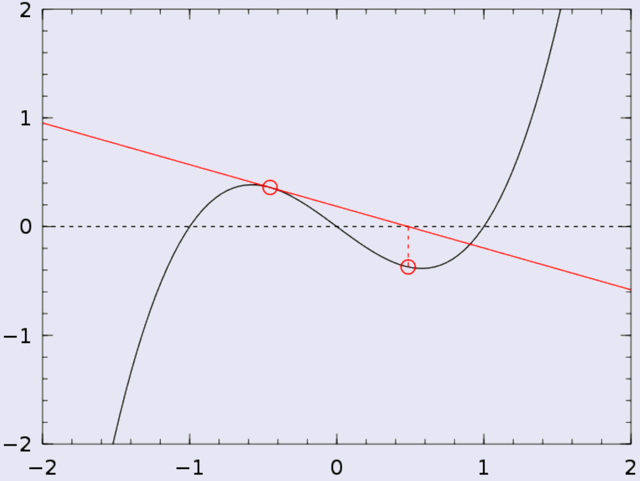

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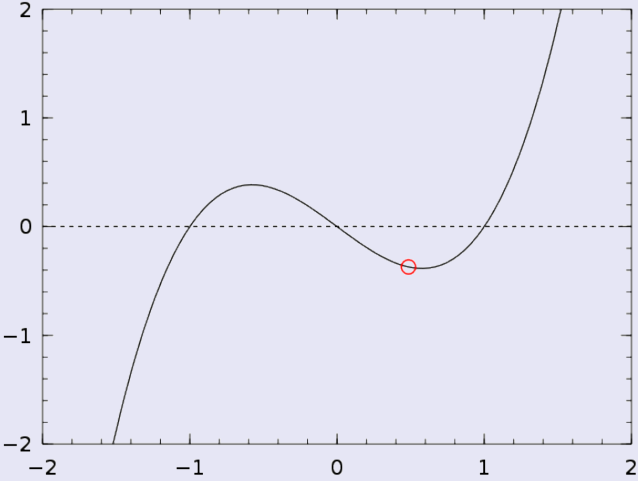

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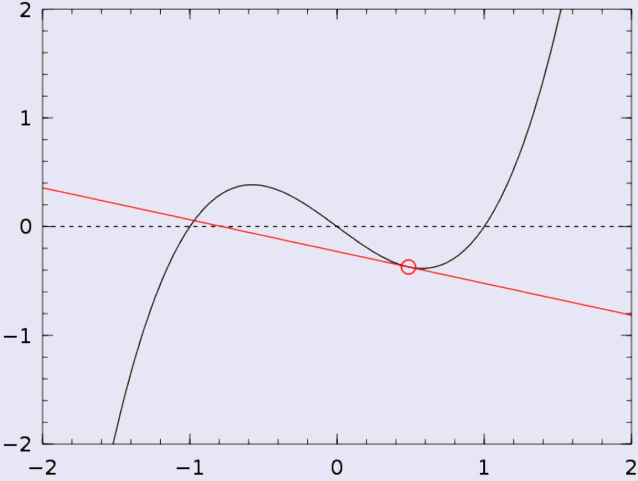

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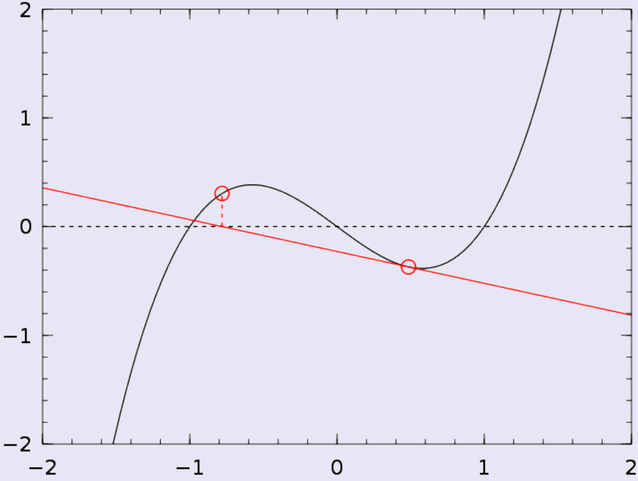

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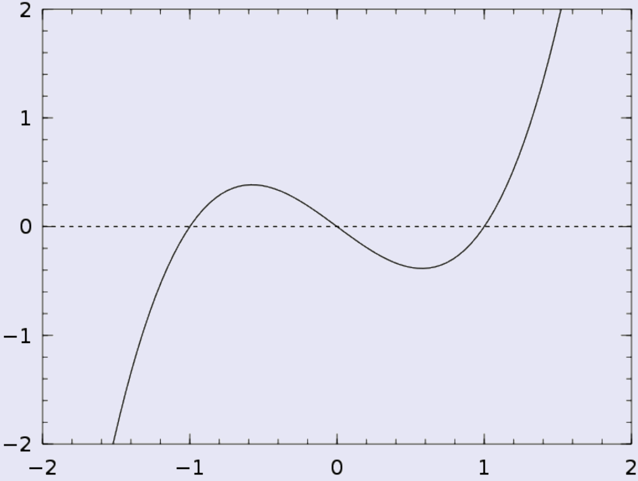

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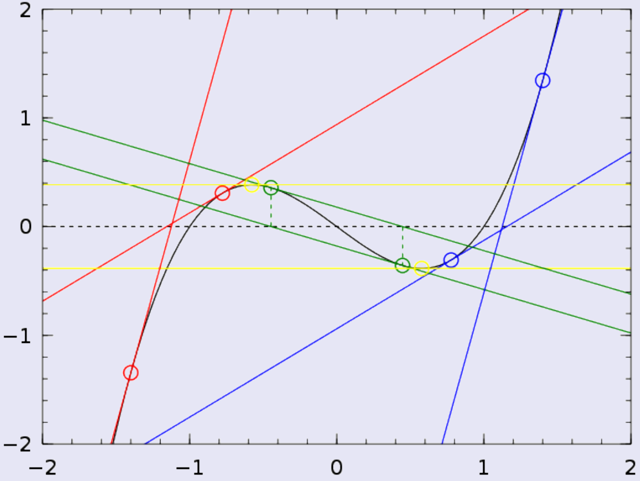

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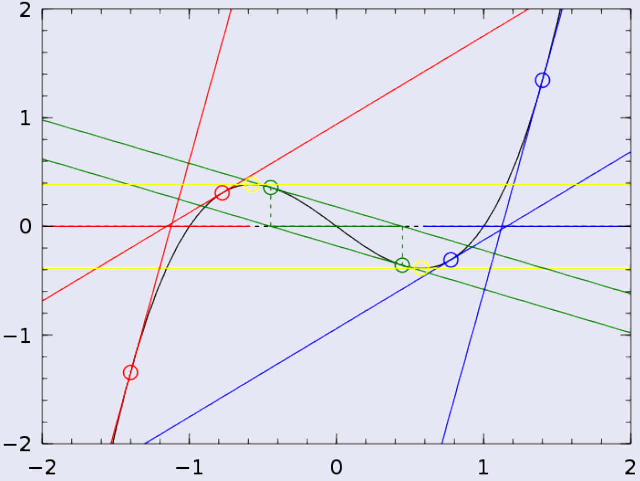

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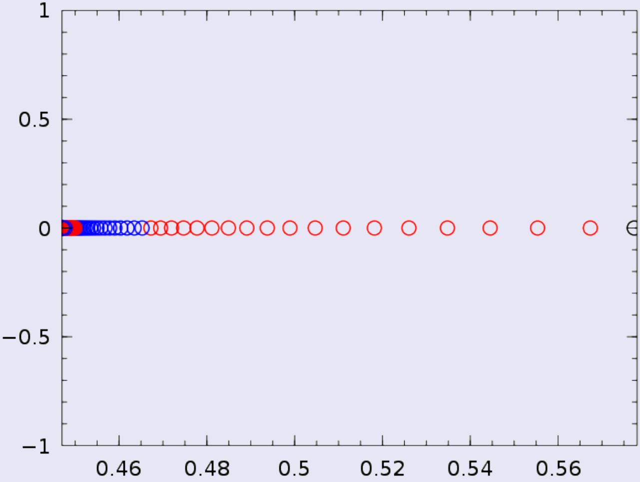

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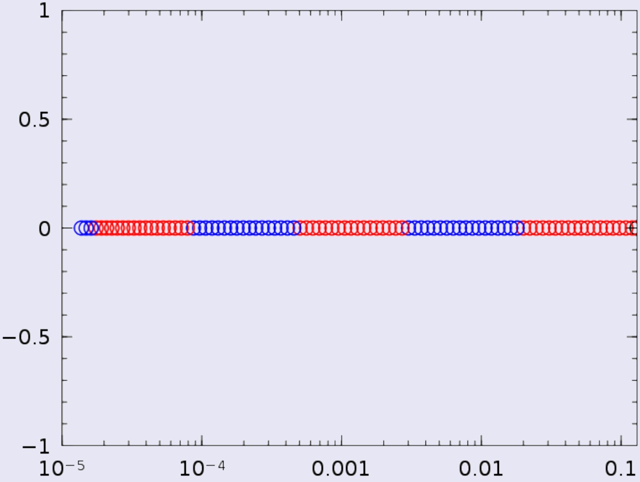
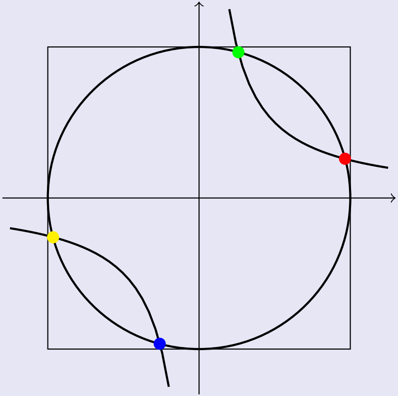
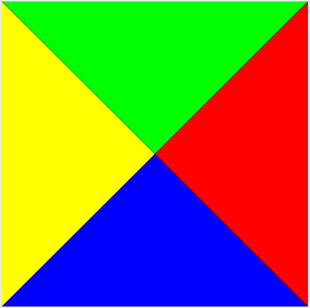


$$\begin{cases} x_1^2 + x_2^2 &= 4 \\ x_1 x_2 &= 1 \end{cases}$$





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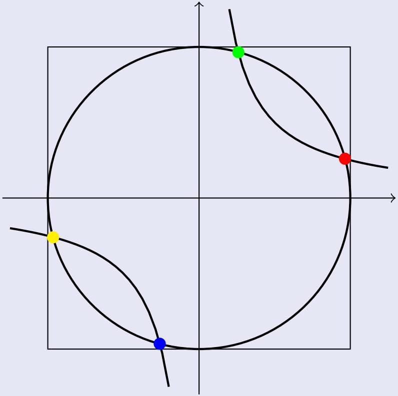

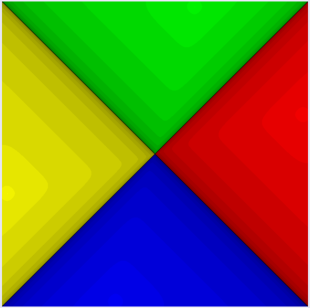

O Exemplo Tradicional

$$z^3 = 1, \qquad z \in \mathbb{C}$$

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$$z = 1, \frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}.$$

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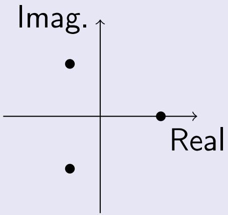

$$z = x + yi$$

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$$z^3 = (x^3 - 3xy^2) + (3x^2y - y^3)i$$

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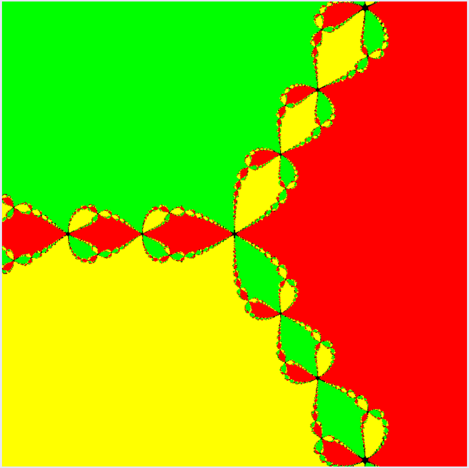
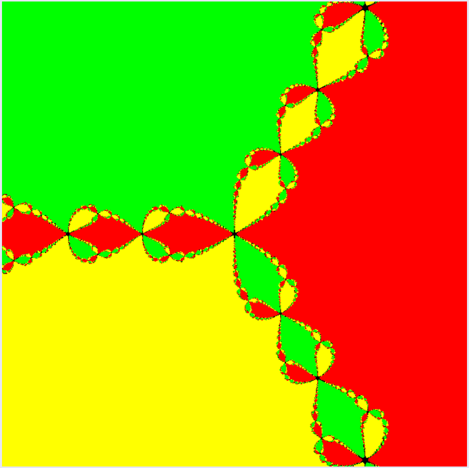
$$F(x,y) = \begin{bmatrix} x^3 - 3xy^2 - 1 \\ 3x^2y - y^3 \end{bmatrix}$$



O fractal de Newton



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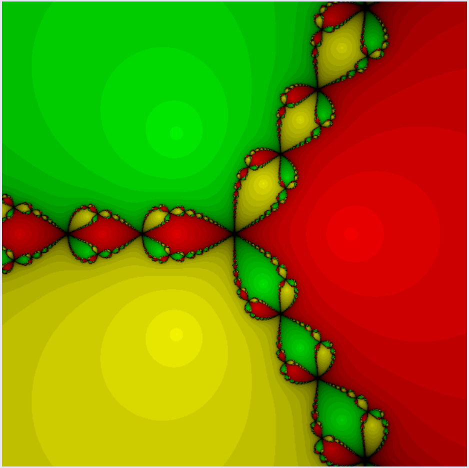
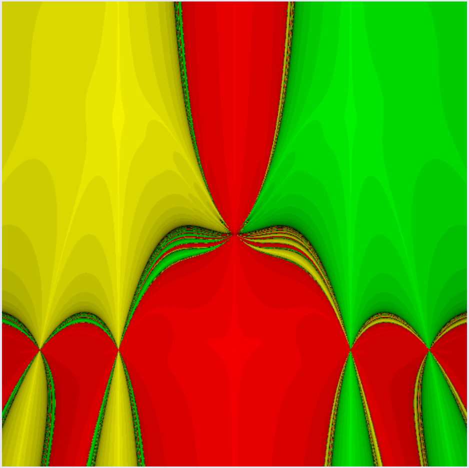
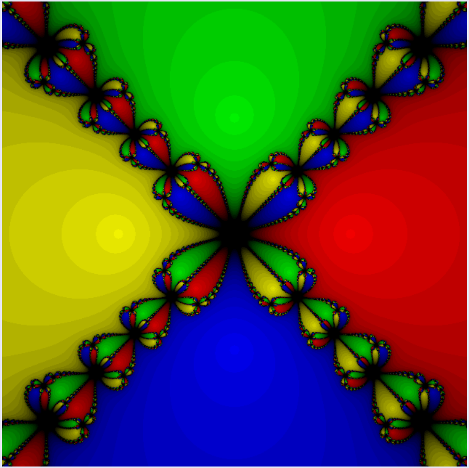


$$F(x, y) = \begin{bmatrix} x(y - x^2) \\ (x^2 - 1)(y + 1) \end{bmatrix}$$



$$z^4 = 1, \qquad z \in \mathbb{C}$$



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Obrigado